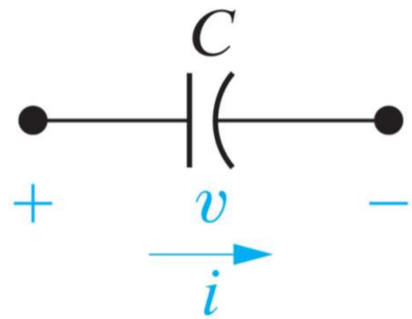
Capacitor



- Parallel plates, separated by an insulator, so no charge flows between the plates. Impose a time-varying voltage drop:
  - → time-varying electric field
  - → time-varying displacement current
- Capacitor equation: $$i(t) = C\frac{dv(t)}{dt}$$
- Units: v(t) is volts, i(t) is amps, and C is farads [F]

Look at the capacitor equation again:

$$i(t) = C\frac{dv(t)}{dt}$$

Suppose *v(t)* is constant. Then *i(t)* =

- ✓ A. 0
- ✗ B. ∞
- ✗ C. a constant

So, if the voltage drop across the capacitor is constant, its current is 0, so the capacitor can be replaced by

- ✗ A. a short circuit
- ✓ B. an open circuit
- ✗ C. a constant

Capacitor

- **If the voltage drop across a capacitor is constant, the current is 0, so the capacitor can be replaced by an OPEN CIRCUIT.**
- Look at the capacitor equation again: $i(t) = C\dfrac{dv(t)}{dt}$
- Suppose there is a discontinuity in *v(t)* – that is, at some value of *t*, the voltage jumps instantaneously. At this value of *t*, the derivative of the voltage is infinite. Therefore the current is infinite! NOT POSSIBLE.
- **Thus, the voltage drop across a capacitor is continuous for all time.**

The voltage drop across a capacitor is described as

$$v(t) = V_o, \quad t < 0$$

$$v(t) = 15e^{-250t} - 10e^{-1000t} \text{ V}, \quad t \geq 0$$

This expression for the capacitor voltage is valid only if the value of V_o is

- ✗ A. 0 V
- ✗ B. 25 V
- ✓ C. 5 V

Capacitor

- The equation for voltage in terms of current:

$$i(t) = C\frac{dv(t)}{dt} \qquad \Rightarrow \qquad i(t)dt = Cdv(t)$$

$$\Rightarrow \qquad \int_{t_o}^{t} i(\tau)d\tau = C\int_{v(t_o)}^{v(t)} dx$$

$$\Rightarrow \qquad v(t) = \frac{1}{C}\int_{t_o}^{t} i(\tau)d\tau + v(t_o)$$

Capacitor

- Power and energy

$$p(t) = v(t)i(t) = Cv(t)\frac{dv(t)}{dt} \qquad \text{(passive sign convention!)}$$

$$p(t) = \frac{dw(t)}{dt} = Cv(t)\frac{dv(t)}{dt}$$

$$\Rightarrow \qquad dw(\tau) = Cv(\tau)dv(\tau)$$

$$\Rightarrow \qquad \int_0^{w(t)} dx = C\int_0^{v(t)} y(\tau)d\tau$$

$$\Rightarrow \qquad w(t) = \tfrac{1}{2}Cv(t)^2$$

Capacitor

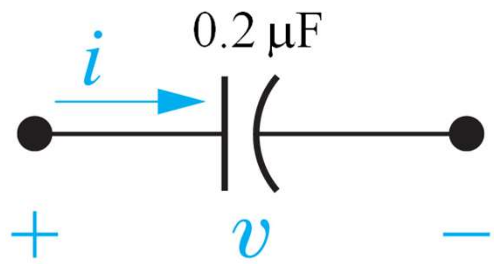
- Capacitors, like inductors, are **energy storage devices**!
  - If the initial voltage drop across the capacitor is non-zero, the capacitor is storing energy.

Suppose the initial voltage drop across a 4 μF capacitor is 10 V. The initial energy stored in the capacitor is

- ✗ A. 400 μJ
- ✓ B. 200 μJ
- ✗ C. 20 μJ

Capacitor

- Example 6.5 – find the voltage, power, and energy



$$i(t) = \begin{cases} 0, & t \le 0; \\ 5000t \text{ A}, & 0 \le t \le 20\ \mu\text{s}; \\ 0.2 - 5000t \text{ A}, & 20 \le t \le 40\ \mu\text{s}; \\ 0, & t \ge 40\ \mu\text{s}. \end{cases}$$

For $t < 0$:

$$v(t) = 0\text{ V}; \qquad p(t) = 0\text{ W}; \qquad w(t) = 0\text{ J}$$

For $0 \le t \le 20\mu\text{s}$:

$$v(t) = \frac{1}{0.2\mu}\int_0^t 5000x\,dx + v(0) = \frac{1}{0.2\mu}\frac{5000x^2}{2}\bigg|_0^t = 12.5\times10^9 t^2\text{ V}$$

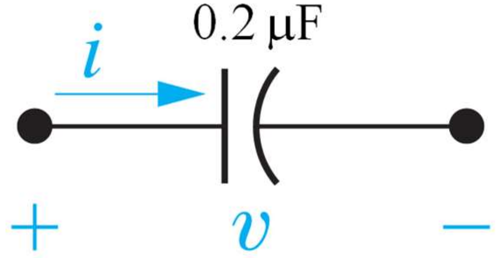
$$p(t) = v(t)i(t) = (12.5\times10^9 t^2)(5000t) = 62.5\times10^{12} t^3\text{ W}$$

$$w(t) = \tfrac{1}{2}(0.2\mu)(12.5\times10^9 t^2)^2 = 15.625\times10^{12} t^4\text{ J}$$

At $t = 20\mu\text{s}$, $\qquad v(20\mu\text{s}) = 12.5\times10^9(20\mu)^2 = 5\text{ V}$

Capacitor

- Example 6.5, continued



$$i(t) = \begin{cases} 0, & t \le 0; \\ 5000t \text{ A}, & 0 \le t \le 20\ \mu\text{s}; \\ 0.2 - 5000t \text{ A}, & 20 \le t \le 40\ \mu\text{s}; \\ 0, & t \ge 40\ \mu\text{s}. \end{cases}$$

For $20\mu\text{s} \le t \le 40\mu\text{s}$:

$$v(t) = \frac{1}{0.2\mu} \int_{20\mu}^{t} (0.2 - 5000x)\, dx + v(20\mu)$$

$$= \frac{1}{0.2\mu} \left[0.2x - \frac{5000x^2}{2} \right]_0^t + 5$$

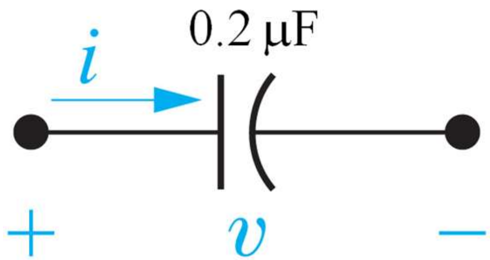
$$= (10^6 t - 12.5 \times 10^9 t^2 - 10)\ \text{V}$$

$$p(t) = v(t)i(t); \qquad w(t) = \tfrac{1}{2} C v(t)^2$$

At $t = 40\mu\text{s}$, $\quad v(40\mu\text{s}) = [10^6(40\mu) - 12.5 \times 10^9 (40\mu)^2 - 10) = 10\ \text{V}$

Capacitor

- Example 6.5, continued



$$i(t) = \begin{cases} 0, & t \le 0; \\ 5000t \text{ A}, & 0 \le t \le 20\ \mu\text{s}; \\ 0.2 - 5000t \text{ A}, & 20 \le t \le 40\ \mu\text{s}; \\ 0, & t \ge 40\ \mu\text{s}. \end{cases}$$

For $t \ge 40\mu\text{s}$:

$$v(t) = \frac{1}{0.2\mu}\int_{40\mu}^{t} 0\,dx + v(40\mu) = 10\text{ V}$$

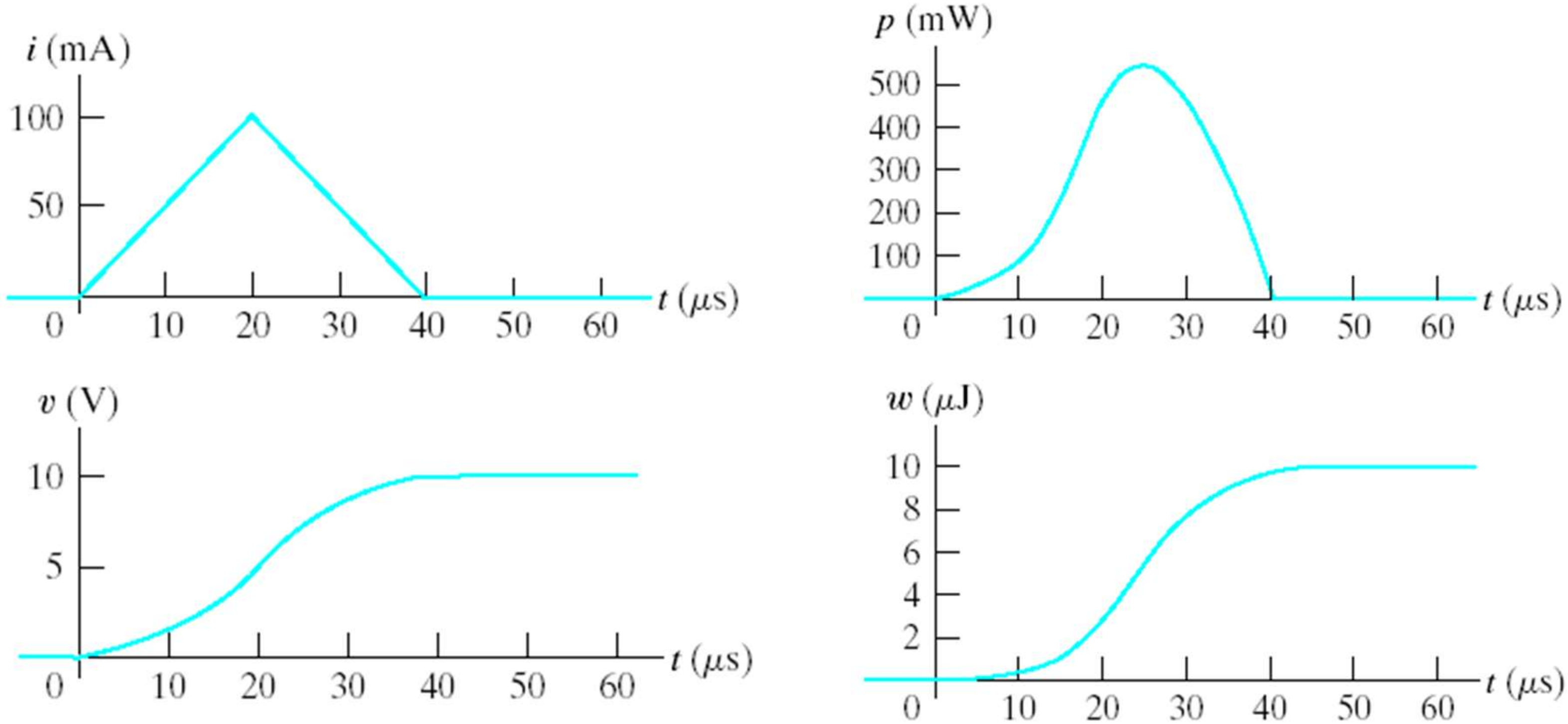
$$p(t) = v(t)i(t) = 0\text{ W}$$

$$w(t) = \tfrac{1}{2}(0.2\mu)(10)^2 = 10\mu\text{J!}$$

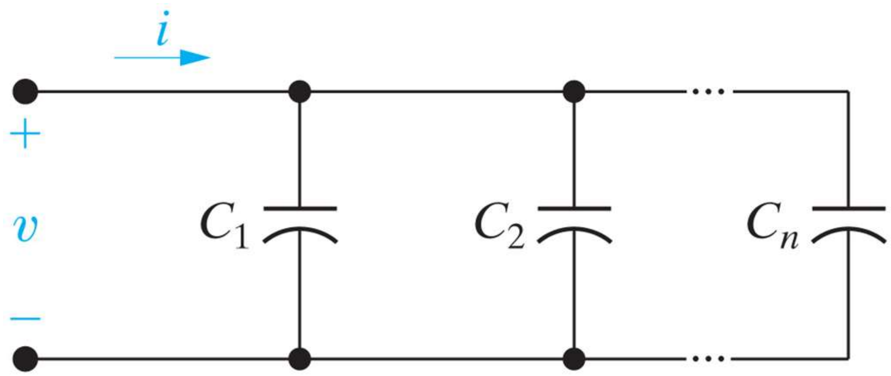
During the interval between 0 and 40μs, the power is positive (absorbed), energy is stored and “trapped” by the capacitor, so even when the current goes to 0, the voltage stays at 10 V and the energy is non-zero.

Capacitor

- Example 6.5, continued



Capacitors in series and parallel

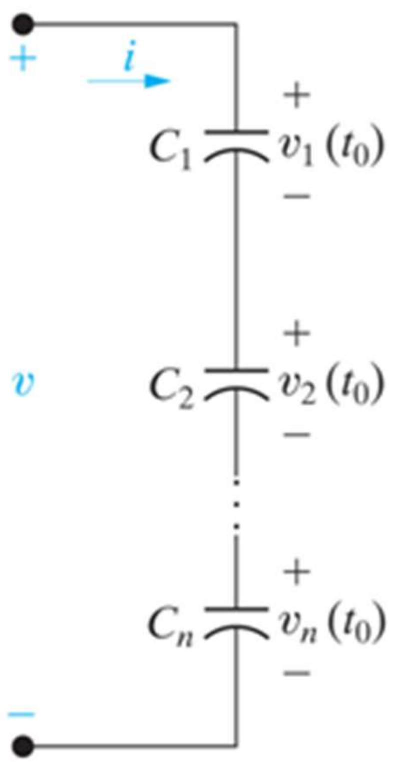


$$\text{KCL}: \qquad i = i_1 + i_2 + i_3 = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + C_3 \frac{dv}{dt}$$

$$= (C_1 + C_2 + C_3) \frac{dv}{dt} = C_{eq} \frac{dv}{dt}$$

Capacitors in parallel ADD.

Capacitors in series and parallel



$$\text{KVL}: \quad v = v_1 + v_2 + v_3 = \frac{1}{C_1}\int_{t_o}^{t} i(\tau)d\tau + v_1(t_o)$$

$$+\frac{1}{C_2}\int_{t_o}^{t} i(\tau)d\tau + v_2(t_o)$$

$$+\frac{1}{C_3}\int_{t_o}^{t} i(\tau)d\tau + v_3(t_o)$$

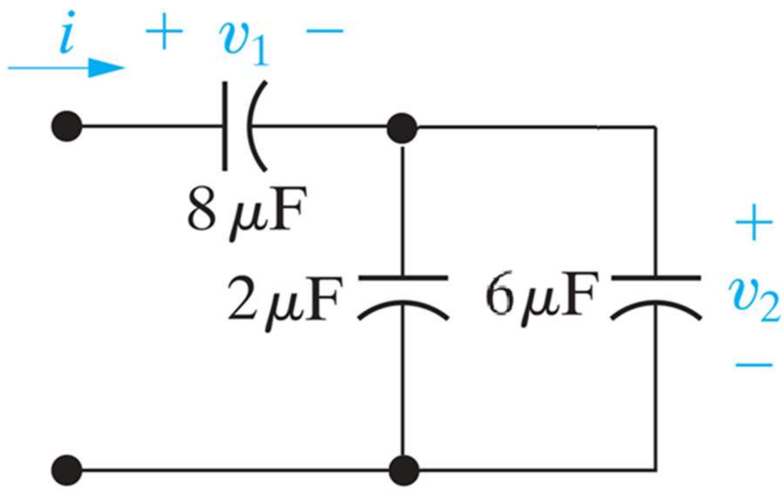
$$= \left(\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}\right)\int_{t_o}^{t} i(\tau)d\tau$$

$$+[v_1(t_o)+v_2(t_o)+v_3(t_o)]$$

$$= \frac{1}{C_{eq}}\int_{t_o}^{t} i(\tau)d\tau + v_{eq}(t_o)$$

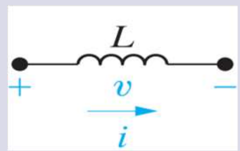
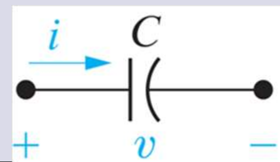
Capacitors combine in series like resistors or inductors combine in parallel.

Find the equivalent capacitance for the circuit below, assuming v_1 = 12 V and v_2 = –8 V.



- A. C_{eq} = 4μF, v_{eq} = 4 V
- B. C_{eq} = 9.5μF, v_{eq} = 4 V
- C. C_{eq} = 4μF, v_{eq} = 20 V

Inductor and Capacitor comparison

	Inductor	**Capacitor**
Symbol		
Units	Henries [H]	Farads [F]
Describing equation	$v(t) = L\frac{di(t)}{dt}$	$i(t) = C\frac{dv(t)}{dt}$
Other equation	$i(t) = \frac{1}{L}\int_{t_o}^{t} v(\tau)d\tau + i(t_o)$	$v(t) = \frac{1}{C}\int_{t_o}^{t} i(\tau)d\tau + v(t_o)$
Initial condition	$i(t_o)$	$v(t_o)$
Behavior with const. source	If $i(t) = I$, $v(t) = 0$ $\rightarrow$ short circuit	If $v(t) = V$, $i(t) = 0$ $\rightarrow$ open circuit
Continuity requirement	$i(t)$ is continuous so $v(t)$ is finite	$v(t)$ is continuous so $i(t)$ is finite

Inductor and Capacitor comparison

	Inductor	Capacitor
Power	$p(t)=v(t)i(t)=Li(t)\dfrac{di(t)}{dt}$	$p(t)=v(t)i(t)=Cv(t)\dfrac{dv(t)}{dt}$
Energy	$w(t)=\frac{1}{2}Li(t)^2$	$w(t)=\frac{1}{2}Cv(t)^2$
Initial energy	$w_o(t)=\frac{1}{2}Li(t_o)^2$	$w_o(t)=\frac{1}{2}Cv(t_o)^2$
Trapped energy	$w(\infty)=\frac{1}{2}Li(\infty)^2$	$w(\infty)=\frac{1}{2}Cv(\infty)^2$
Series-connected	$L_{eq}=L_1+L_2+L_2$ $i_{eq}(t_o)=i(t_o)$	$\dfrac{1}{C_{eq}}=\dfrac{1}{C_1}+\dfrac{1}{C_2}+\dfrac{1}{C_3}$ $v_{eq}(t_o)=v_1(t_o)+v_2(t_o)+v_3(t_o)$
Parallel-connected	$\dfrac{1}{L_{eq}}=\dfrac{1}{L_1}+\dfrac{1}{L_2}+\dfrac{1}{L_3}$ $i_{eq}(t_o)=i_1(t_o)+i_2(t_o)+i_3(t_o)$	$C_{eq}=C_1+C_2+C_2$ $v_{eq}(t_o)=v(t_o)$