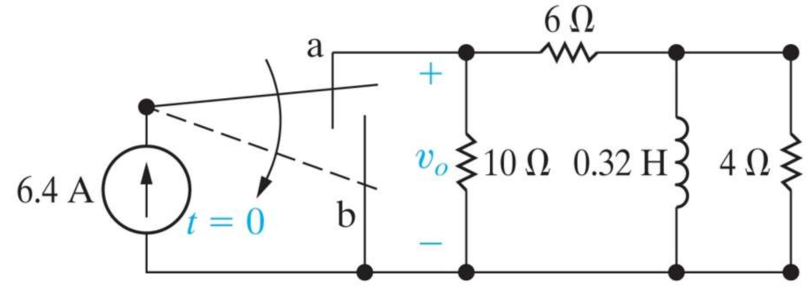
RL/RC Natural Response Summary

1. Identify the variable of interest.
   - For RL, *i(t)* through L; For RC, *v(t)* across C
2. Find the initial value of this variable, either $i(0) = I_o$ or $v(0) = V_o$. Usually, analyze the circuit for t < 0.
3. Find the time constant, τ
   - $\tau_{RL} = L/R_{eq}$ or $\tau_{RC} = R_{eq}C$
   - R_{eq} is the equivalent resistance seen by the inductor or capacitor.
4. Write the expression for the variable of interest:
   $x(t) = X_o e^{-t/\tau}, \quad t \geq 0.$
5. Use simple circuit analysis to calculate any other requested variables.

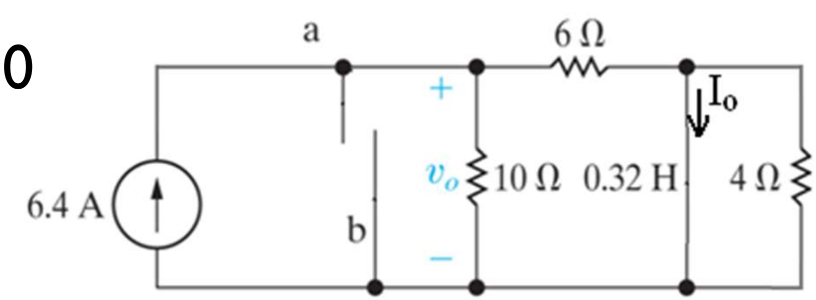
RL/RC Natural Response Examples

AP 7.2 – Find $v_o(t)$ for $t \geq 0^+$

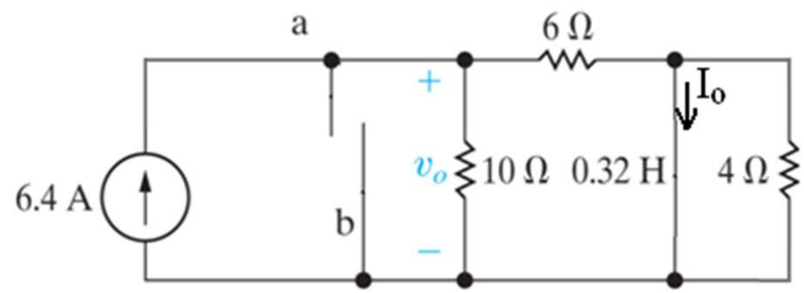


1. Variable of interest is the current through the inductor, so mark it on the circuit.
2. Find the initial current in the inductor:

For $t < 0$



The short circuit that replaced the inductor effectively eliminates which resistor from this circuit?

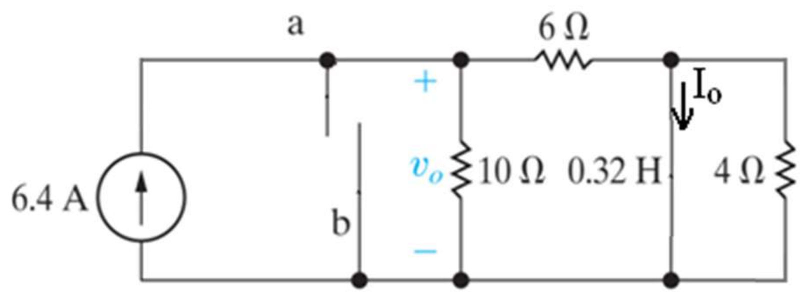


A. 10 Ω

B. 6 Ω

C. 4 Ω

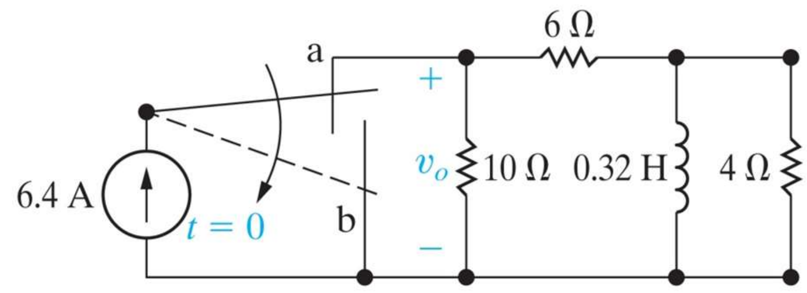
What circuit analysis tool provides the most direct method to calculate I_o?



A. Node voltage method
B. Current division
C. Source transformation

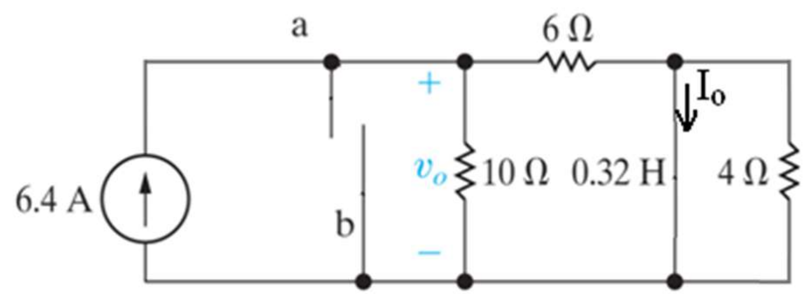
RL/RC Natural Response Examples

AP 7.2 – Find $v_o(t)$ for $t \geq 0^+$



2. Find the initial current in the inductor:

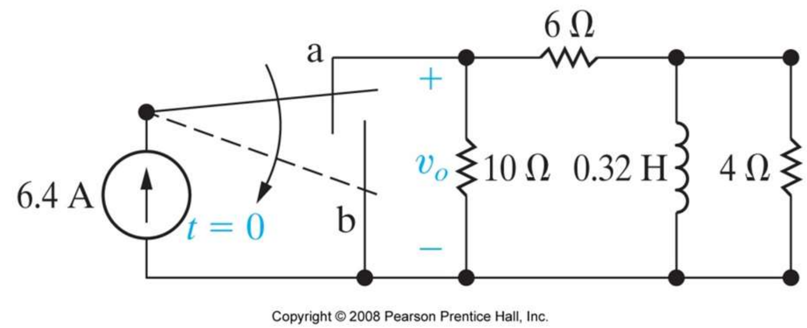
For t < 0



Current division: $$I_o = \frac{10}{10+6}(6.4) = 4\text{ A}$$

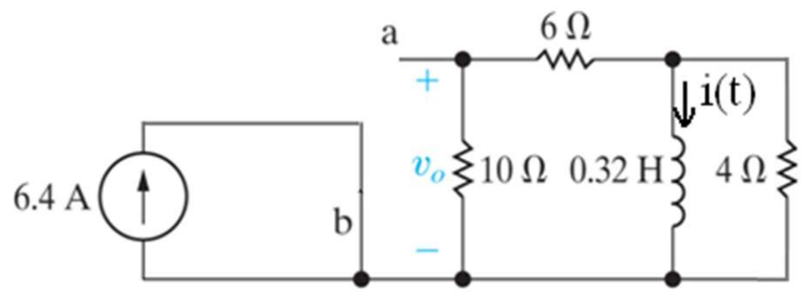
RL/RC Natural Response Examples

AP 7.2 – Find $v_o(t)$ for $t \geq 0^+$

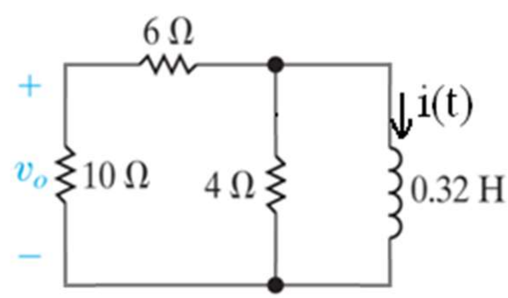


3. Find the time constant, $\tau = L/R_{eq}$:

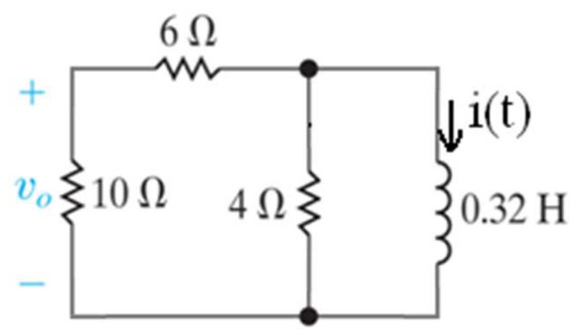
For $t \geq 0$



Simplified:



The expression for the equivalent resistance seen by the inductor is



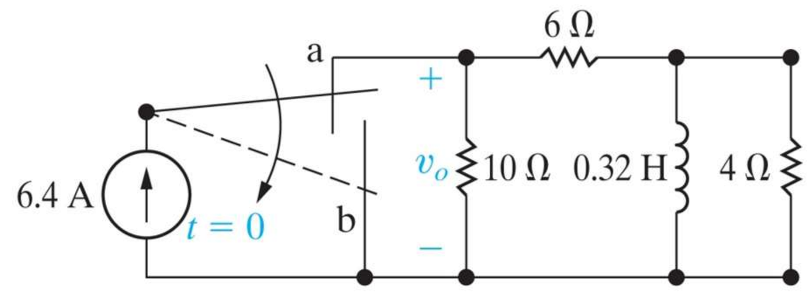
A. 4 || (10 + 6)

B. 10 || (6 + 4)

C. 10 + 6 + 4

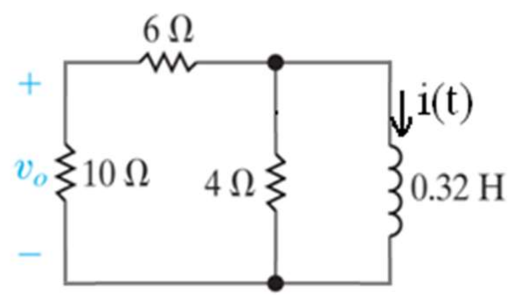
RL/RC Natural Response Examples

AP 7.2 – Find $v_o(t)$ for $t \geq 0^+$



3. Find the time constant, $\tau = L/R_{eq}$:

For $t \geq 0$



$$R_{eq} = 4 \,||\, (6+10) = 3.2\Omega$$

$$\tau = L / R_{eq} = 0.32 / 3.2 = 0.1\,\mathrm{s}$$

4. $i(t) = ICe^{-t/\tau} = 4e^{-t/0.1} = 4e^{-10t}$ A, $t \geq 0$
5. Find $v_o(t)$:

$$v_L(t) = (0.32)\frac{d}{dt}(4e^{-10t}) = (0.32)(-10)(4e^{-10t})$$

$$= -12.8e^{-10t}\ \mathrm{V}, \quad t \geq 0^+$$

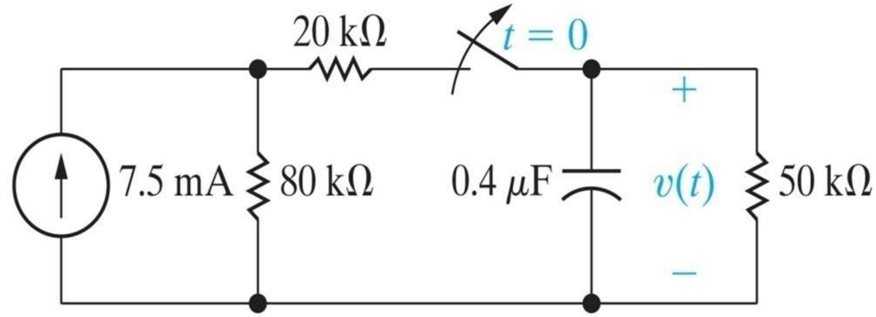
$$v_o(t) = \frac{10}{10+6} v_L(t) = -8e^{-10t}\ \mathrm{V}, \quad t \geq 0^+$$

In circuits with inductors, we solve for the inductor current and the solution is valid for $t \geq 0$. When we use the inductor current to solve for any other voltage or current in the circuit, the resulting expression is valid for $t \geq 0^+$ because none of these other quantities are required to be continuous everywhere.

A. True
B. False

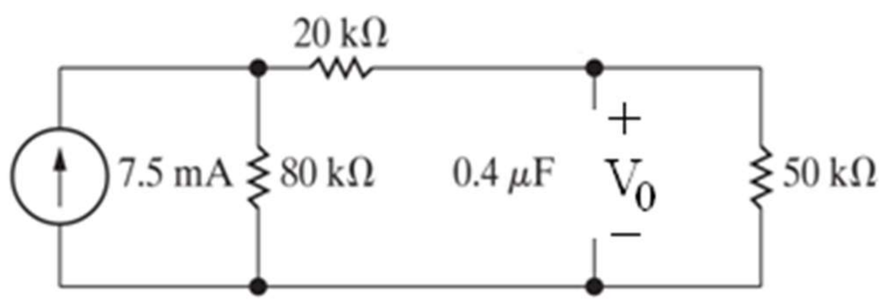
RL/RC Natural Response Examples

AP 7.3 – Find the time at which 75% of the initial stored energy has been dissipated.

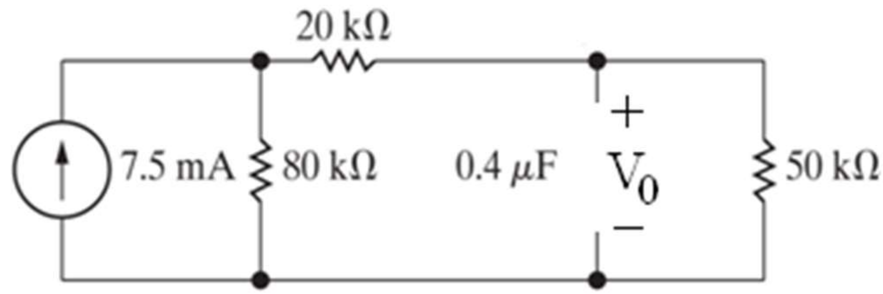


1. Variable of interest is the voltage drop across the capacitor, which is already marked on the circuit.
2. Find the initial voltage drop across the capacitor:

t < 0:



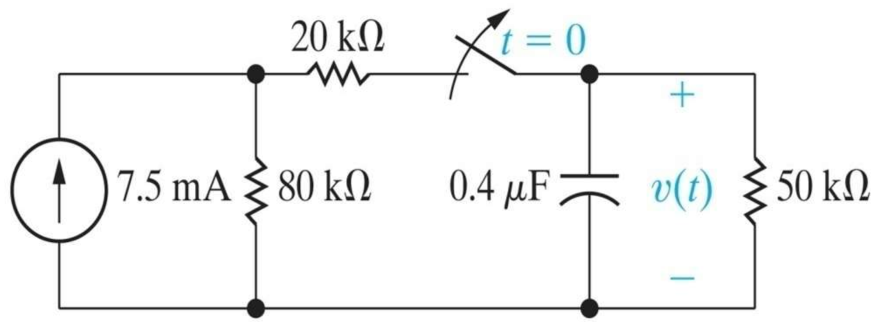
In the circuit used to find the initial voltage drop across the capacitor, the voltage drop across the open circuit is the same as the voltage drop across



A. The 80 kΩ resistor
B. The 50 kΩ resistor
C. The 20 kΩ resistor

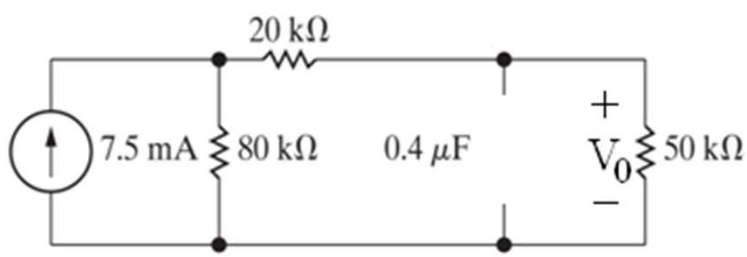
RL/RC Natural Response Examples

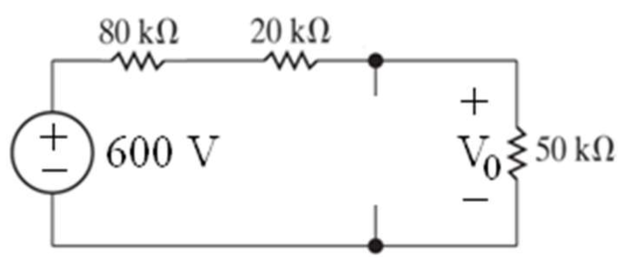
AP 7.3 – Find the time at which 75% of the initial stored energy has been dissipated.



2. Find the initial voltage drop across the capacitor:

t < 0:

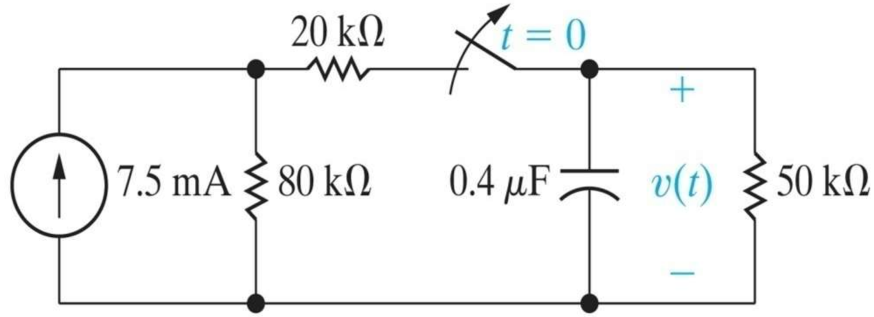




$$V_0 = \frac{50\,\text{k}}{80\,\text{k} + 20\,\text{k} + 50\,\text{k}}(600)$$
$$= 200\ \text{V}$$

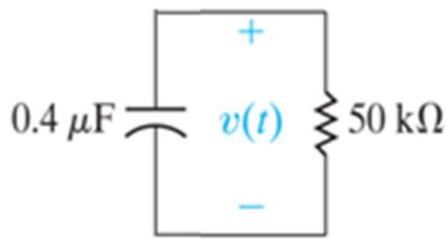
RL/RC Natural Response Examples

AP 7.3 – Find the time at which 75% of the initial stored energy has been dissipated.



3. Find the time constant, $\tau = R_{eq}C$

$t \geq 0$:

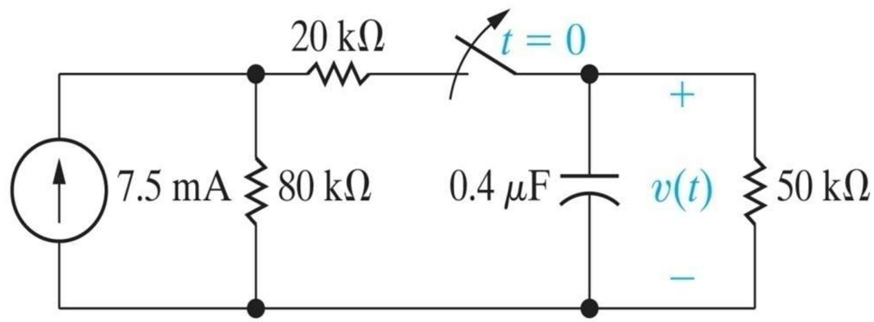


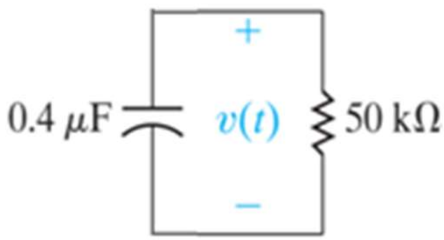
$$\tau = (50{,}000)(0.4 \times 10^{-6}) = 0.02\text{ s}$$

4. $v(t) = ICe^{-t/\tau} = 200e^{-t/0.02} = 200e^{-50t}\,\text{V},\ t \geq 0$

RL/RC Natural Response Examples

AP 7.3 – Find the time at which 75% of the initial stored energy has been dissipated.





$$v(t) = 200e^{-50t} \text{ V}, \quad t \geq 0$$

5. Find the initial energy. $w(0) = \frac{1}{2}(0.4\times10^{-6})(200)^2 = 8 \text{ mJ}$

   75% of the initial energy is 6 mJ, so the energy that remains is 2 mJ. Thus,

$$w(t) = \tfrac{1}{2}(0.4\times10^{-6})(200e^{-50t})^2 = 8e^{-100t} \text{ mJ} = 2 \text{ mJ}$$

$$\Rightarrow e^{-100t} = 0.25 \qquad \Rightarrow t = \frac{\ln(0.25)}{-100} = 13.86 \text{ ms}$$