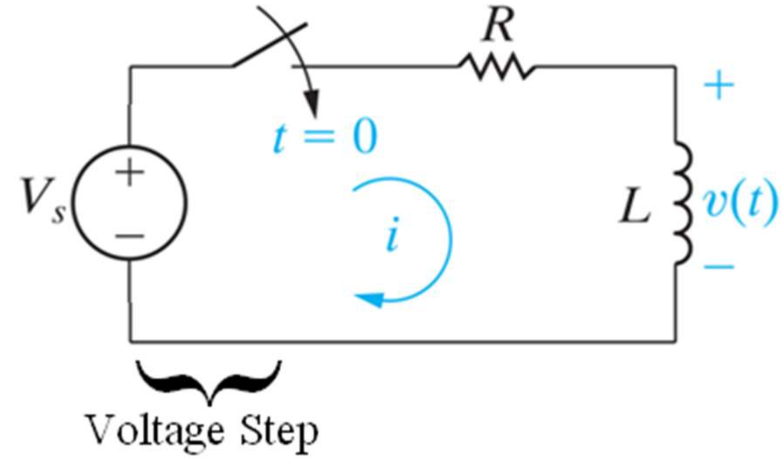
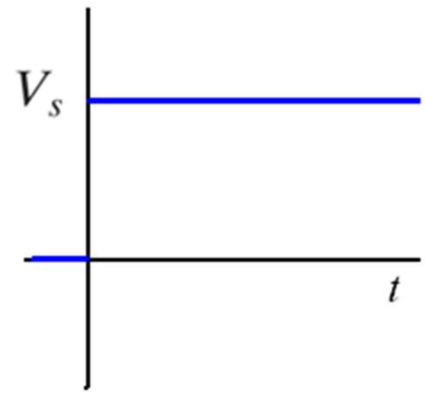
# RL Step Response

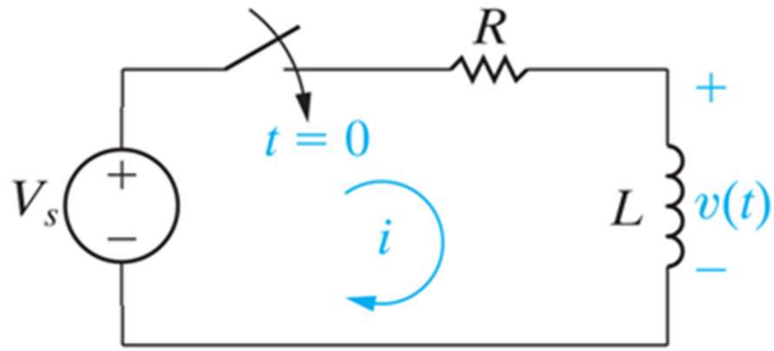
The problem: Given an initial current *i(0)* through *L* at $t = 0$, find $i(t)$ for $t \geq 0$. Note that the initial current may be 0!




This is the step response of this circuit because for $t \geq 0$ there is an independent source in the circuit – the response is due to the initial energy stored in the inductor as well as the energy supplied by the voltage source for $t \geq 0$ .

# RL Step Response




KVL for $t \geq 0$:

$$-V_S + L\frac{di(t)}{dt} + Ri(t) = 0$$

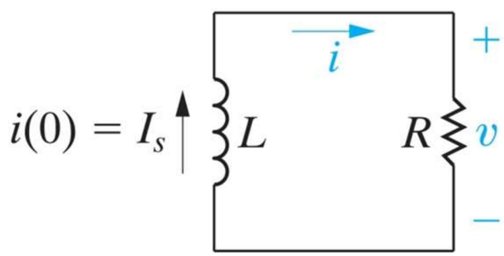
$$\Rightarrow \qquad L\frac{di(t)}{dt} + Ri(t) = V_S$$

This equation is a

- First-order (the highest derivative is a first derivative)
- Inhomogeneous (the right-hand side is non-zero)
- Ordinary differential equation
- With constant coefficients

The left-hand side of the equation is identical to the describing differential equation for the RL Natural Response!

The circuit we analyzed for the RL natural response problem is shown below, along with the expression for *i(t)*, for $t \geq 0$. What is the value of *i(t)* as $t \to \infty$?



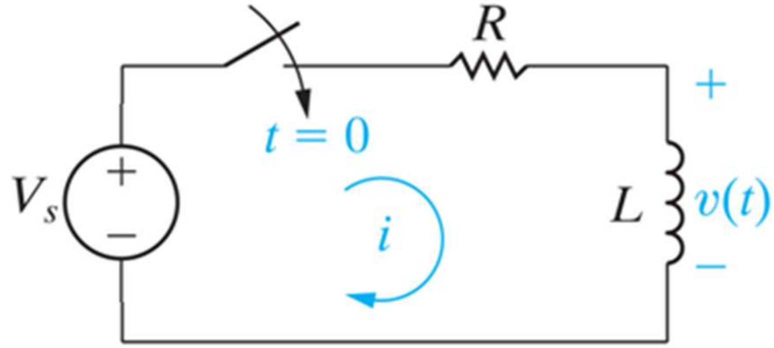



$$i(t) = I_s e^{-(R/L)t}, \qquad t \geq 0$$

A. *0*

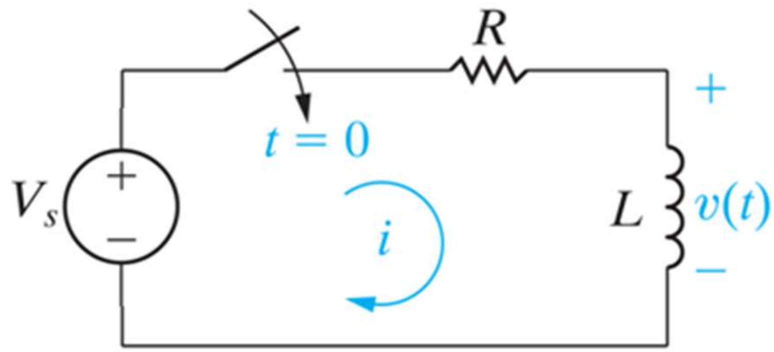
B. $I_S$

C. ∞




Now consider what happens to the current as $t \to \infty$ in the RL step response circuit. First, as $t \to \infty$, we can replace the inductor by

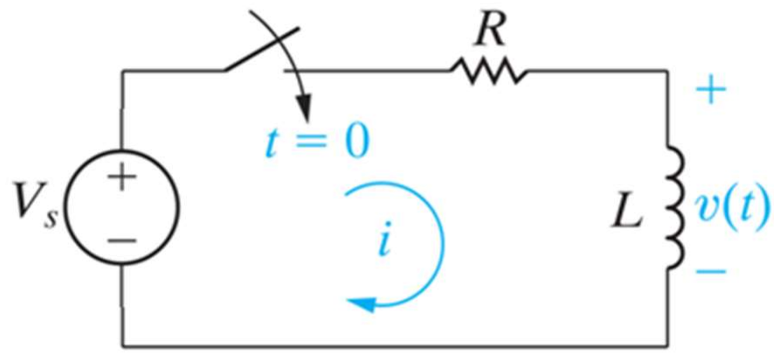
A. An open circuit
B. A short circuit
C. A resistor




To evaluate the current in the inductor as $t \to \infty$, close the switch and replace the inductor with a short circuit. The current in the short circuit is

A. 0
B. infinite
C. $V_S$ /R

# RL Step Response




KVL: $$L\frac{di(t)}{dt} + Ri(t) = V_S$$

The functional form for $i(t)$ should be the same as for the natural response!

Try $\quad i(t) = I_0 e^{-(R/L)t}$, $\quad$ where $I_0$ is the initial inductor current.

But as $t \to \infty$ $\quad i(\infty) = I_0 e^{-(R/L)(\infty)} = 0$ $\quad$ which is incorrect!

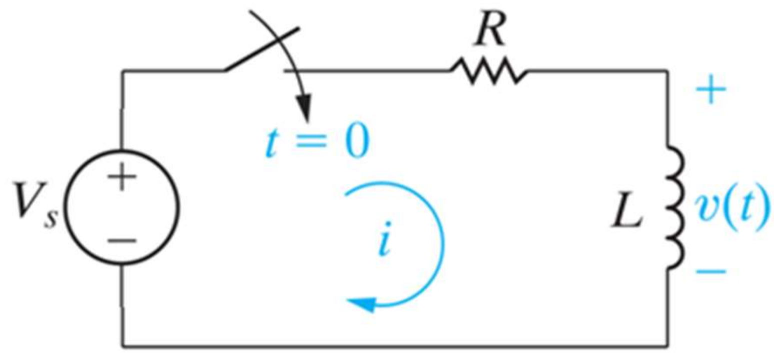
Try $\quad i(t) = I_F + I_0 e^{-(R/L)t}$,

where $I_F$ is the inductor current as $t \to \infty$.

Then $\quad i(\infty) = I_F + I_0 e^{-(R/L)(\infty)} = I_F$ $\quad$ (OK)

But $\quad i(0) = I_F + I_0 e^{-(R/L)(0)} = I_F + I_0$ $\quad$ (NO!)

# RL Step Response




$$\text{KVL}: \qquad L\frac{di(t)}{dt} + Ri(t) = V_S$$

Now try $\quad i(t) = I_F + (I_0 - I_F)e^{-(R/L)t}$

Then $\quad i(\infty) = I_F + (I_0 - I_F)e^{-(R/L)(\infty)} = I_F \qquad$ (OK)

And $\quad i(0) = I_F + (I_0 - I_F)e^{-(R/L)(0)} = I_F + I_0 - I_F = I_0 \qquad$ (OK)

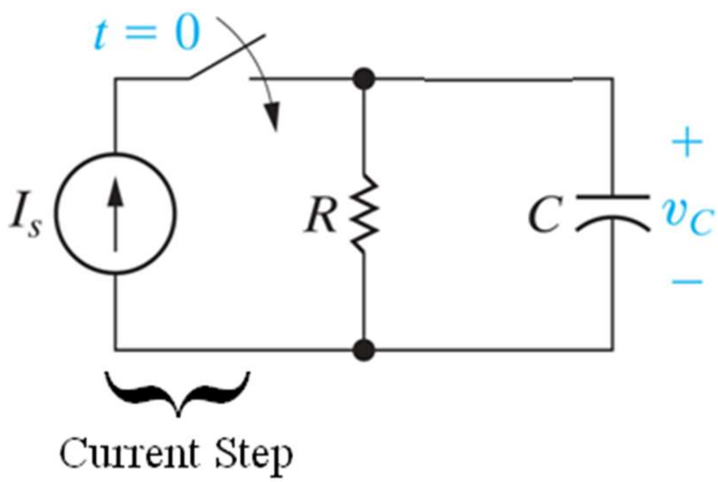
Make sure that this $i(t)$ satisfies the differential equation :

$$i(t) = I_F + (I_0 - I_F)e^{-(R/L)t}; \qquad \frac{di(t)}{dt} = -(R/L)(I_0 - I_F)e^{-(R/L)t}$$

Substituting,

$$\mathrm{L}[-(R/L)(I_0 - I_F)e^{-(R/L)t}] + R[I_F + (I_0 - I_F)e^{-(R/L)t}] = RI_F = V_S$$

# RC Step Response

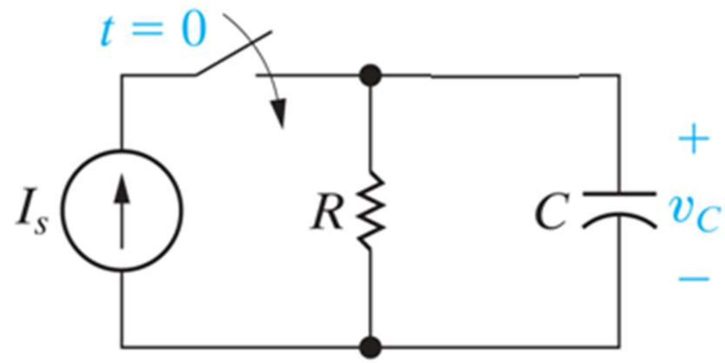



The problem: Given an initial voltage drop *v(0)* across *C* at *t = 0*, find *v(t)* for *t ≥ 0*. Note that the initial voltage may be 0!

This is the duel of the RL step response problem, so its solution is the duel of the solution to the RL step response!

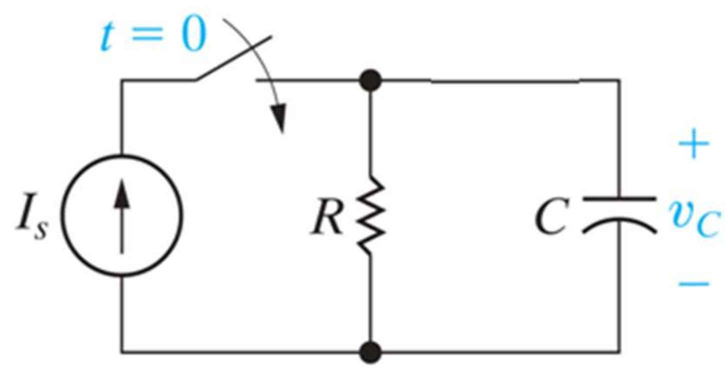
$$v_C(t) = V_F + (V_0 - V_F)e^{-t/RC}, \qquad t \geq 0$$

$$\text{where} \qquad V_0 = v_C(0) \qquad \text{and} \qquad V_F = v_C(\infty)$$




Here is the circuit for the RC step response problem. As $t \to \infty$, the capacitor can be replaced by

A. An open circuit
B. A short circuit
C. A constant current




To evaluate the voltage drop across the capacitor as $t \to \infty$, close the switch and replace the capacitor with an open circuit. The voltage drop across the open circuit, positive at the top, is

A. 0
B. $I_S R$
C. infinite

# RL/RC Step (and Natural) Response

A general solution method:

1. Identify the variable of interest (hint – it's the variable that must be continuous in the circuit):
    - For RL, *i(t)* through L
    - For RC, *v(t)* across C
2. Find the initial value of this variable, either $i(0) = I_o$ or $v(0) = V_o$
    - From the problem statement
    - By analyzing the circuit for $t < 0$, with L replaced by a short circuit or C replaced by an open circuit

# RL/RC Step (and Natural) Response

A general solution method, continued:

3. Find the final value of this variable, either $i(\infty) = I_F$ or $v(\infty) = V_F$
   - If there is no source in the circuit as $t \rightarrow \infty$, the final value is 0 – this is the natural response problem!
   - Otherwise, analyze the circuit as $t \rightarrow \infty$, with L replaced by a short circuit or C replaced by an open circuit
4. Find the time constant, $\tau$
   - $\tau_{RL} = L/R_{eq}$ or $\tau_{RC} = R_{eq}C$
   - Note that $R_{eq}$ is the equivalent resistance as seen from the inductor or capacitor for $t \geq 0$

# RL/RC Step (and Natural) Response

A general solution method, continued:

5. Write the expression for the variable of interest:

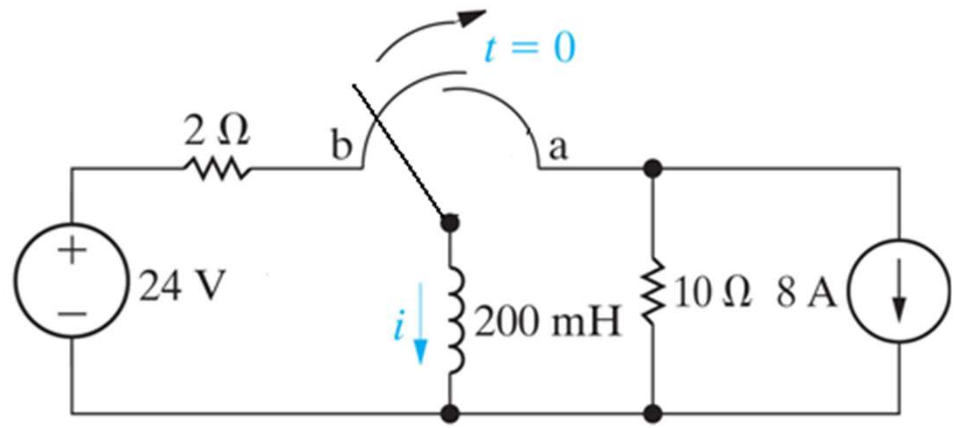
   $x(t) = X_F + (X_o - X_F)e^{-t/\tau}, \quad t \geq 0$

6. Use simple circuit analysis to calculate any other requested variables.

# RL/RC Step Response Examples

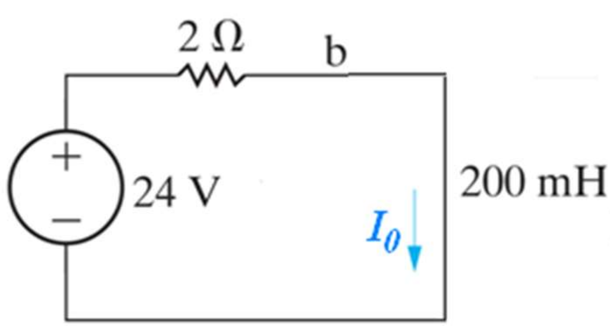
AP 7.5

Find i(t) for $t \geq 0$




1. Variable of interest is the current through the inductor, which is already indicated in the drawing.
2. Find the initial current in the inductor:

For $t < 0$

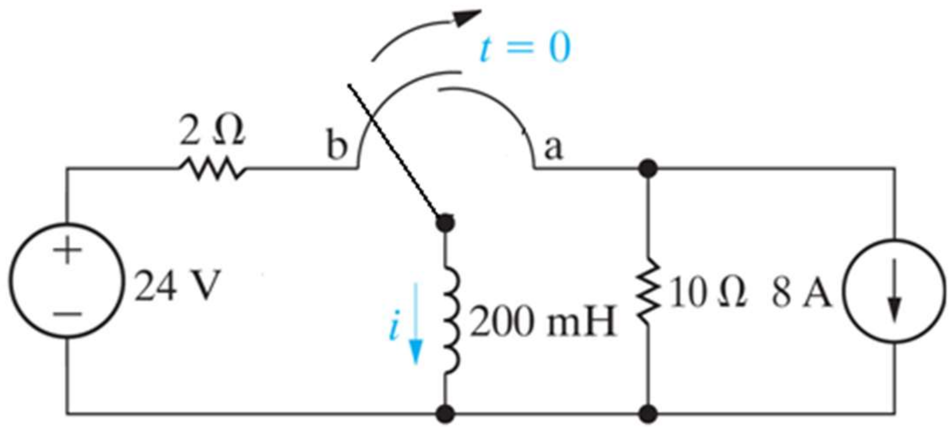



$$I_0 = \frac{24}{2} = 12\text{ A}$$

# RL/RC Step Response Examples

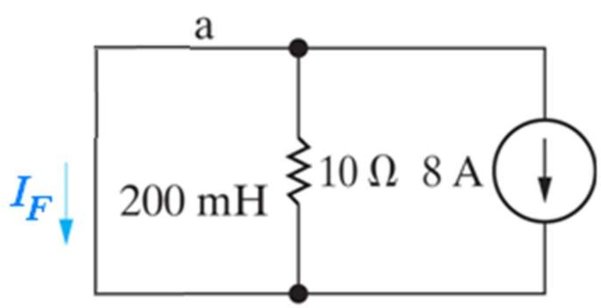
AP 7.5

Find i(t) for t ≥ 0



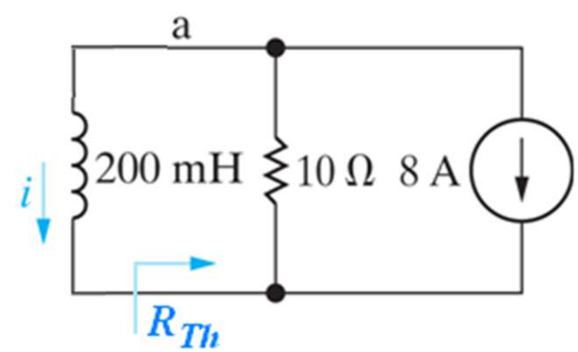

3. Find the final current in the inductor:

As $t \to \infty$




$$I_F = -8 \text{ A}$$

4. Find the time constant, $\tau = L/R_{eq}$, for $t \geq 0$. What is the Thevenin equivalent resistance of the circuit attached to the inductor?

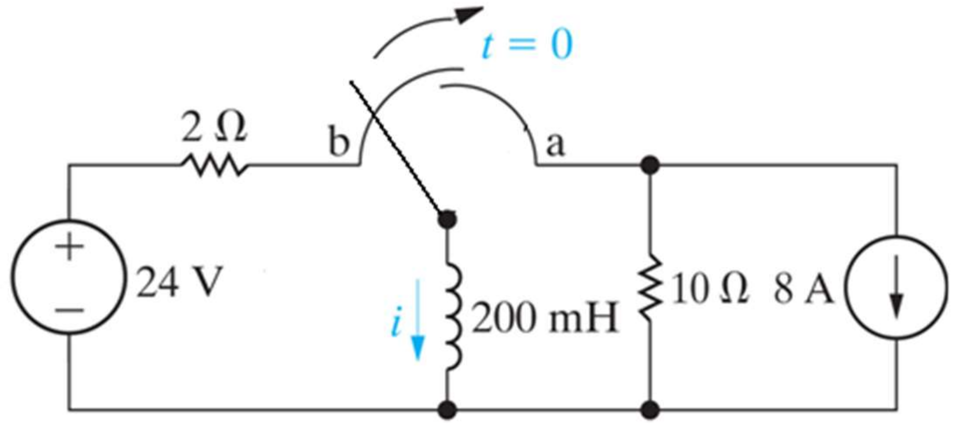



A. 10 Ω
B. 80 Ω
C. 200 Ω

# RL/RC Step Response Examples

AP 7.5

Find i(t) for t ≥ 0




4. The time constant is $\tau$ = *(0.2)/(10)* = *0.02* s.
5. Write the expression for the inductor current:

$$i(t) = I_F + (I_0 - I_F)e^{-t/\tau}$$

$$= -8 + [12 - (-8)]e^{-t/0.02}$$

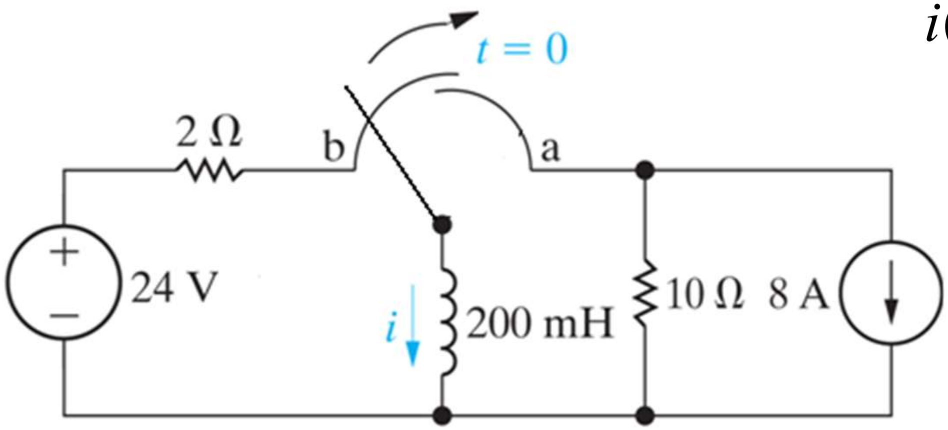
$$= -8 + 20e^{-50t} \text{ A}, \quad t \geq 0$$

$$\text{Check :} \qquad i(0) = -8 + 20e^{-50(0)} = -8 + 20 = 12 \text{ A} \ \ (\text{OK})$$

$$i(\infty) = -8 + 20e^{-50(\infty)} = -8 \text{ A} \ \ (\text{OK})$$

Find the current in the resistor (flowing from top to bottom), given the inductor current just calculated, for $t \geq 0$.

$$i(t) = -8 + 20e^{-50t}\ \text{A}, \quad t \geq 0$$

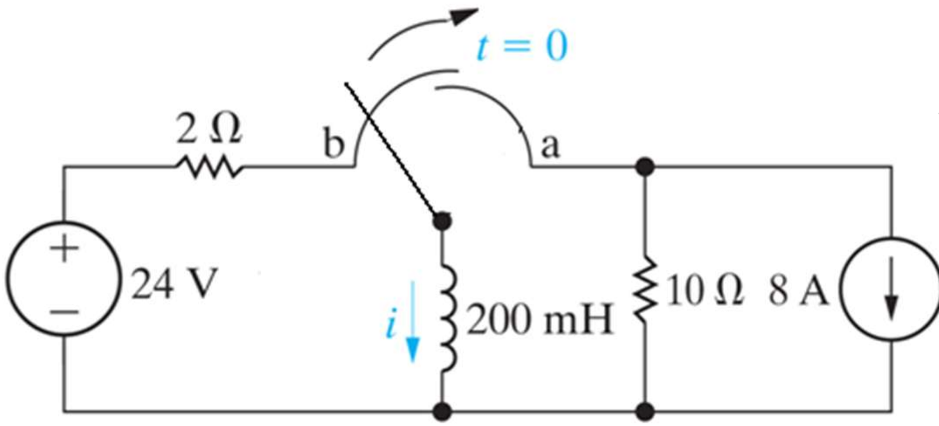



A. $-8 + 20e^{-50t}$ A, $t \geq 0$

B. $20e^{-50t}$ A, $t \geq 0$

C. $-20e^{-50t}$ A, $t \geq 0$

Find the voltage drop across the resistor (positive at the top), given the resistor current just calculated, for $t \geq 0^+$.




$$i_{10\Omega}(t) = -20e^{-50t}\ \text{A}, \quad t \geq 0$$

A. $-2e^{-50t}$ V, $t \geq 0^+$

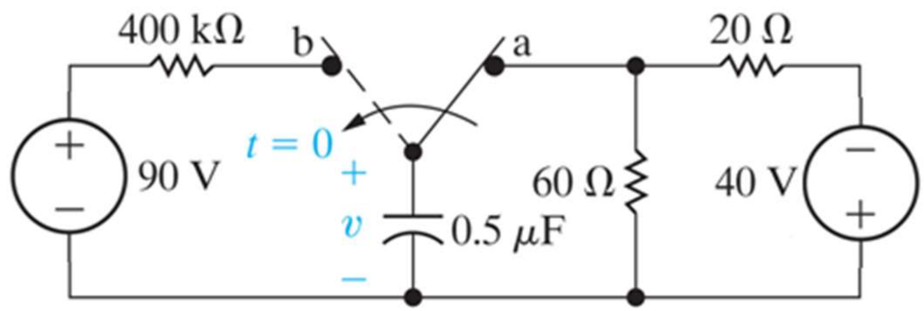
B. $200e^{-50t}$ V, $t \geq 0^+$

C. $-200e^{-50t}$ V, $t \geq 0^+$

# RL/RC Step Response Examples

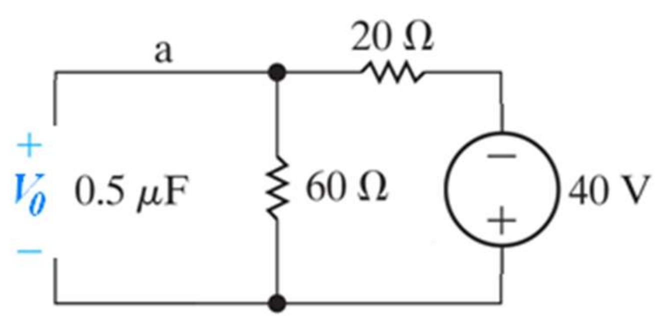
Ex 7.7

Find *v(t)* for *t* ≥ *0*

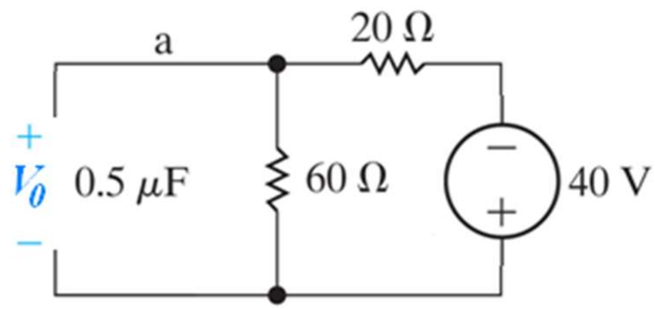



1. The variable of interest is the capacitor voltage drop, which is already defined in the circuit.
2. Find the initial voltage drop across the capacitor:

For t < 0



What is the best circuit analysis tool to use when finding the initial voltage drop across the capacitor?

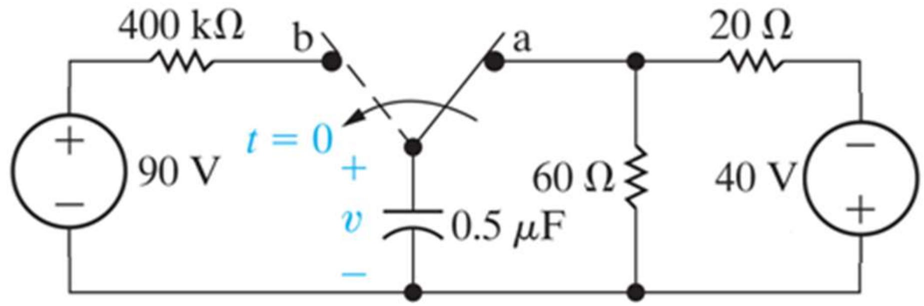



A. Current division
B. Voltage division
C. Source transform

# RL/RC Step Response Examples

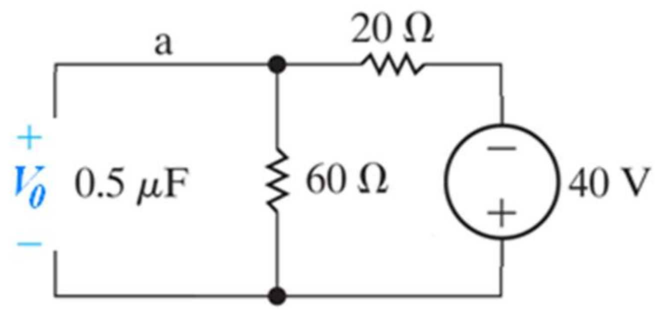
Ex 7.7

Find *v(t)* for *t* ≥ *0*




2. Find the initial voltage drop across the capacitor:

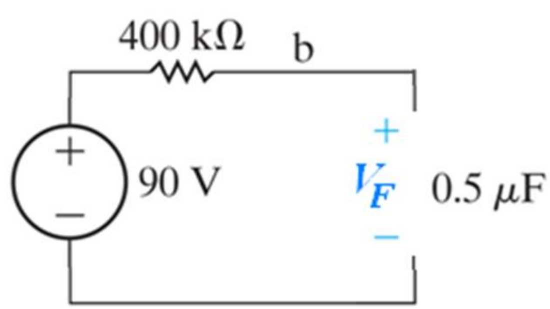
For t < 0




$$V_o = \frac{60}{60+20}(-40) = -30\ \text{V}$$

3. Find the final voltage drop across the capacitor:

As $t \to \infty$

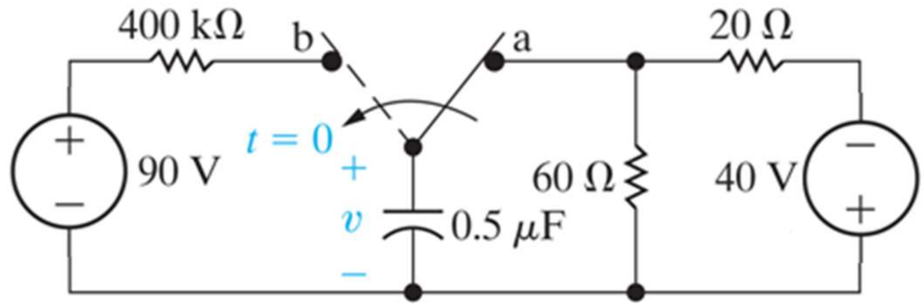



$$V_F = 90\ \text{V}$$

# RL/RC Step Response Examples

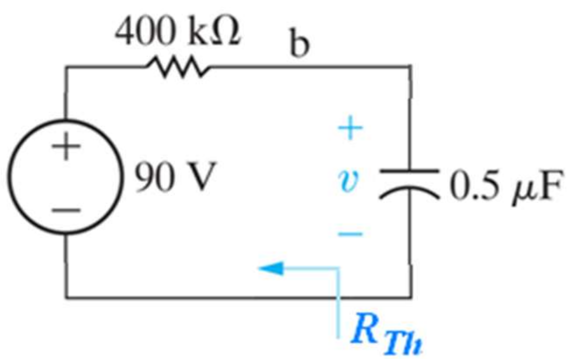
Ex 7.7

Find *v(t)* for *t* ≥ *0*




4. Find the time constant, $\tau = R_{eq}C$ by finding the equivalent resistance seen by the capacitor for $t \geq 0$.

For $t \geq 0$




$$R_{Th} = 400\ \mathrm{k\Omega}$$

$$\tau = (400{,}000)(0.5 \times 10^{-6}) = 0.2\ \mathrm{s}$$

5. Write the expression for the inductor current:

$$v(t) = V_F + (V_0 - V_F)e^{-t/\tau} = 90 + [-30 - 90]e^{-t/0.2}$$

$$= 90 - 120e^{-5t}\ \mathrm{V}, \quad t \geq 0 \qquad (\text{check at } t = 0 \text{ and } t \to \infty)$$

# RL/RC Natural Response Summary

1. Identify the variable of interest.
   - For RL, $i(t)$ through L; For RC, $v(t)$ across C
2. Find the initial value of this variable, either $i(0) = I_o$ or $v(0) = V_o$. Usually, analyze the circuit for t < 0.
3. Find the final value of this variable, either $i(\infty) = I_F$ or $v(\infty) = V_F$. Usually, analyze the circuit as $t \to \infty$.
4. Find the time constant, $\tau$
   - $\tau_{RL} = L/R_{eq}$ or $\tau_{RC} = R_{eq}C$
   - $R_{eq}$ is the equivalent resistance seen by the inductor or capacitor.
5. Write the expression for the variable of interest:
   $$x(t) = X_F + (X_0 - X_F)e^{-t/\tau}, \quad t \geq 0.$$
6. Use simple circuit analysis to calculate any other requested variables.