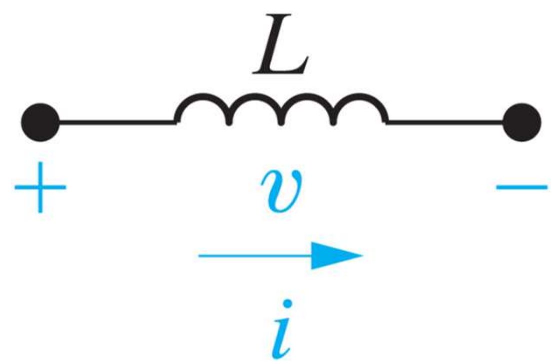
Chapter 6 – Inductance and Capacitance

- Objectives:
  - Inductor
    - Equations for v, i, p, and w
    - Behavior in the presence of constant current
    - Continuity requirements
    - Combine in series and in parallel
  - Capacitor
    - Equations for v, i, p, and w
    - Behavior in the presence of constant voltage
    - Continuity requirements
    - Combine in series and in parallel

Inductor



- Coil of wire with time-varying current
  $\rightarrow$ time-varying magnetic field
  $\rightarrow$time-varying voltage drop

- Inductor equation: $$v(t) = L\frac{di(t)}{dt}$$

- Units: v(t) is volts, i(t) is amps, and L is henries [H]

Look at the inductor equation again:

$$v(t) = L\frac{di(t)}{dt}$$

Suppose i(t) is constant. Then v(t) =

✗ A. L
✓ B. 0
✗ C. Undefined

So, if the current in an inductor is constant, its voltage drop is 0, so the inductor can be replaced by

✔ A. A short circuit

✘ B. An open circuit

✘ C. A capacitor

Inductor

- **If the current in the inductor is constant, the voltage is 0, so the inductor can be replaced by a SHORT CIRCUIT.**
- Look at the inductor equation again: $v(t) = L\frac{di(t)}{dt}$
- Suppose there is a discontinuity in i(t) – that is, at some value of t, the current jumps instantaneously. At this value of t, the derivative of the current is infinite. Therefore the voltage is infinite! NOT POSSIBLE.
- **Thus, the current through an inductor is continuous for all time.**

We just showed that the current in an inductor must be continuous for all time. This means that the voltage must also be continuous for all time.

A. True
B. False

Inductor

- The equation for current in terms of voltage:

$$v(t) = L\frac{di(t)}{dt} \qquad \Rightarrow \qquad v(t)dt = Ldi(t)$$

$$\Rightarrow \qquad \int_{t_o}^{t} v(\tau)d\tau = L\int_{i(t_o)}^{i(t)} dx$$

$$\Rightarrow \qquad i(t) = \frac{1}{L}\int_{t_o}^{t} v(\tau)d\tau + i(t_o)$$

Inductor

- Power and energy

$$p(t) = v(t)i(t) = Li(t)\frac{di(t)}{dt} \qquad \text{(passive sign convention!)}$$

$$p(t) = \frac{dw(t)}{dt} = Li(t)\frac{di(t)}{dt}$$

$$\Rightarrow \qquad dw(\tau) = Li(\tau)di(\tau)$$

$$\Rightarrow \qquad \int_0^{w(t)} dx = L\int_0^{i(t)} y(\tau)d\tau$$

$$\Rightarrow \qquad w(t) = \tfrac{1}{2}Li(t)^2$$

Inductor

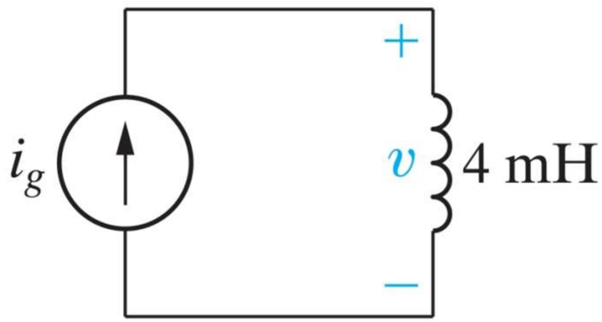
- Inductors are **energy storage devices**!
  - If the initial current through the inductor is non-zero, the inductor is storing energy.

Suppose the initial current through a 2 mH inductor is 1 A. The initial energy stored in the inductor is

- ✓ A. 1 mJ
- ✗ B. 2 mJ
- ✗ C. 4 mJ

Inductor

- Example – Assessment Problem 6.1



$$i_g(t) = 0, \qquad t < 0,$$

$$i_g(t) = 8e^{-300t} - 8e^{-1200t}\ \text{A}, \qquad t \geq 0.$$

Is the current continuous? i(0) = 8 – 8 = 0: YES!

Find the voltage:

$$v(t) = 0, \qquad t < 0$$

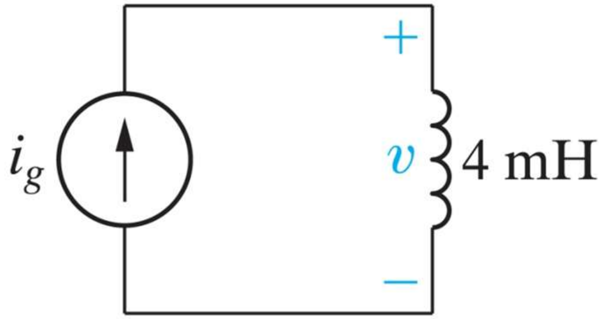
$$v(t) = L\frac{di(t)}{dt} = (0.004)[(-300)8e^{-300t} - (-1200)8e^{-1200t}]$$

$$= -9.6e^{-300t} + 38.4e^{-1200t}\ \text{V}, \qquad t \geq 0$$

Is the voltage continuous? v(0) = -9.6 + 38.4 = 28.8 V: NO!

Inductor

- Example – Assessment Problem 6.1, continued



$$i_g(t) = 0, \qquad t < 0,$$

$$i_g(t) = 8e^{-300t} - 8e^{-1200t}\ \text{A}, \qquad t \geq 0.$$

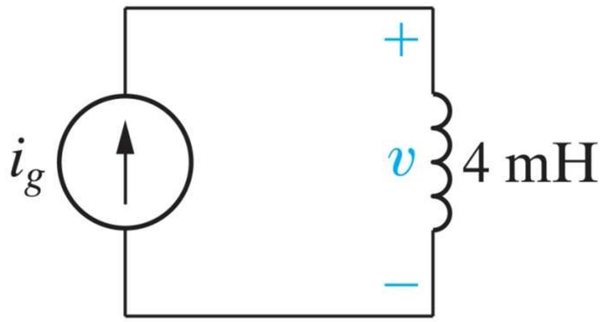
Find the power for the inductor:

$$\begin{aligned} p(t) &= v(t)i(t) \\ &= (-9.6e^{-300t} + 38.4e^{-1200t})(8e^{-300t} - 8e^{-1200t}) \\ &= -76.8e^{-600t} + 384e^{-1500t} - 307.2e^{-2400t}\ \text{W} \end{aligned}$$

To find the max power and the time at which the power is max, take the first derivative of p(t) and set it equal to 0.

Inductor

- Example – Assessment Problem 6.1, continued



$$i_g(t) = 0, \qquad t < 0,$$

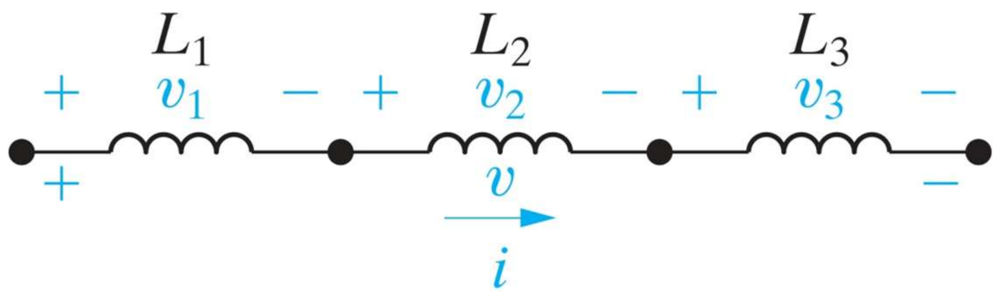
$$i_g(t) = 8e^{-300t} - 8e^{-1200t}\ \text{A}, \qquad t \geq 0.$$

Find the energy for the inductor:

$$\begin{aligned} w(t) &= \tfrac{1}{2} L i(t)^2 \\ &= \tfrac{1}{2}(0.004)(8e^{-300t} - 8e^{-1200t})^2 \\ &= 128(e^{-600t} - 2e^{-1500t} + e^{-2400t})\ \text{mJ} \end{aligned}$$

To find the max energy and the time at which the energy is max, take the first derivative of w(t) and set it equal to 0. Or if you don't have w(t) yet, do the same for i(t)!

Inductors in series and parallel



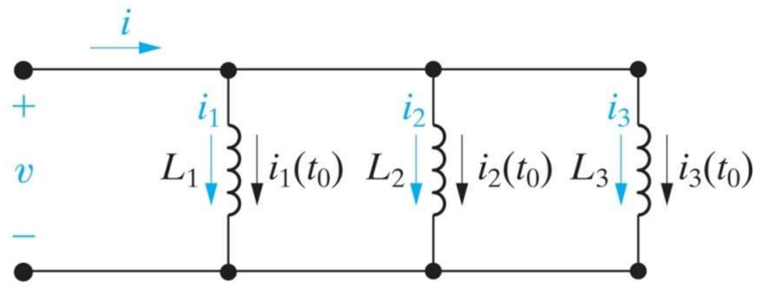
$$v_1 = L_1\frac{di}{dt}, \qquad v_2 = L_2\frac{di}{dt}, \qquad \text{and} \qquad v_3 = L_3\frac{di}{dt}.$$

KVL:

$$v = v_1 + v_2 + v_3 = (L_1 + L_2 + L_3)\frac{di}{dt},$$

Inductors in series ADD.

Inductors in series and parallel



$$i_1 = \frac{1}{L_1}\int_{t_0}^{t} v\,d\tau + i_1(t_0), \quad i_2 = \frac{1}{L_2}\int_{t_0}^{t} v\,d\tau + i_2(t_0), \quad i_3 = \frac{1}{L_3}\int_{t_0}^{t} v\,d\tau + i_3(t_0).$$

KCL: $i = i_1 + i_2 + i_3.$

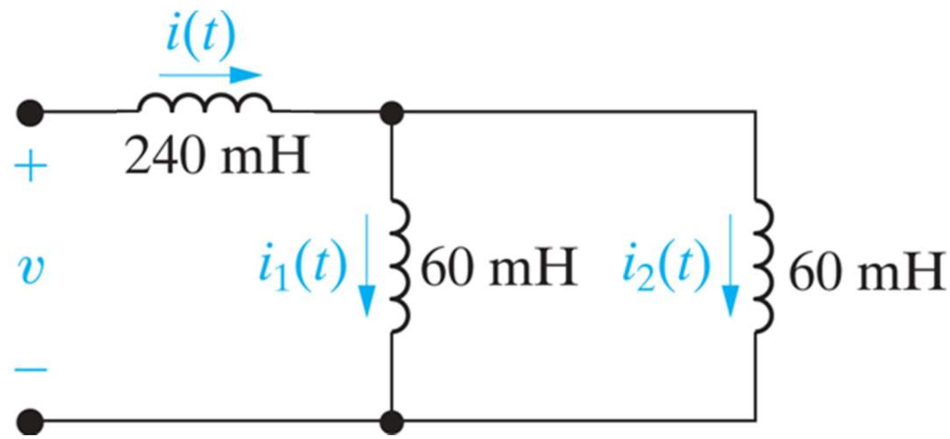
$$i = \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right)\int_{t_0}^{t} v\,d\tau + i_1(t_0) + i_2(t_0) + i_3(t_0).$$

$$i = \frac{1}{L_{eq}}\int_{t_0}^{t} v\,d\tau + i(t_0).$$

Inductors combine in parallel like resistors combine in parallel.

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \qquad i(t_0) = i_1(t_0) + i_2(t_0) + i_3(t_0)$$

Find the equivalent inductance for the circuit below, assuming i_1= 6 A and i_2 = -3 A.

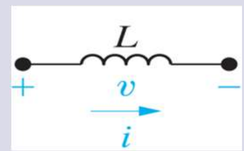


A. L_{eq} = 300 mH, i(t) = 6 A

B. L_{eq} = 270 mH, i(t) = -3 A

C. L_{eq} = 270 mH, i(t) = 3 A

Inductor summary

Symbol	
Units	Henries [H]
Describing equation	$v(t) = L\frac{di(t)}{dt}$
Other equation	$i(t) = \frac{1}{L}\int_{t_o}^{t} v(\tau)d\tau + i(t_o)$
Initial condition	$i(t_o)$
Behavior with const. source	If $i(t) = I$, $v(t) = 0 \rightarrow$ short circuit
Continuity requirement	$i(t)$ is continuous so $v(t)$ is finite

Inductor summary

Power	$p(t)=v(t)i(t)=Li(t)\frac{di(t)}{dt}$
Energy	$w(t)=\frac{1}{2}Li(t)^2$
Initial energy	$w_o(t)=\frac{1}{2}Li(t_o)^2$
Trapped energy	$w(\infty)=\frac{1}{2}Li(\infty)^2$
Series-connected	$L_{eq}=L_1+L_2+L_2$ $i_{eq}(t_o)=i(t_o)$
Parallel-connected	$\frac{1}{L_{eq}}=\frac{1}{L_1}+\frac{1}{L_2}+\frac{1}{L_3}$ $i_{eq}(t_o)=i_1(t_o)+i_2(t_o)+i_3(t_o)$