Basic Laws of Electric Circuits

Kirchhoff’s Voltage Law

Basic Laws of Circuits

<u>Kirchhoff's Voltage Law</u>:

- **Kirchhoff's voltage law tells us how to handle voltages in an electric circuit.**

- **Kirchhoff's voltage law basically states that the algebraic sum of the voltages around any closed path (electric circuit) equal zero. The secret here, as in Kirchhoff's current law, is the word algebraic.**

- **There are three ways we can interrupt that the algebraic sum of the voltages around a closed path equal zero. This is similar to what we encountered with Kirchhoff's current law.**

Basic Laws of Circuits

Kirchhoff's Voltage Law:

Consideration 1: **Sum of the voltage drops around a circuit equal zero. We first define a drop.**

We assume a circuit of the following configuration. Notice that no current has been assumed for this case, at this point.

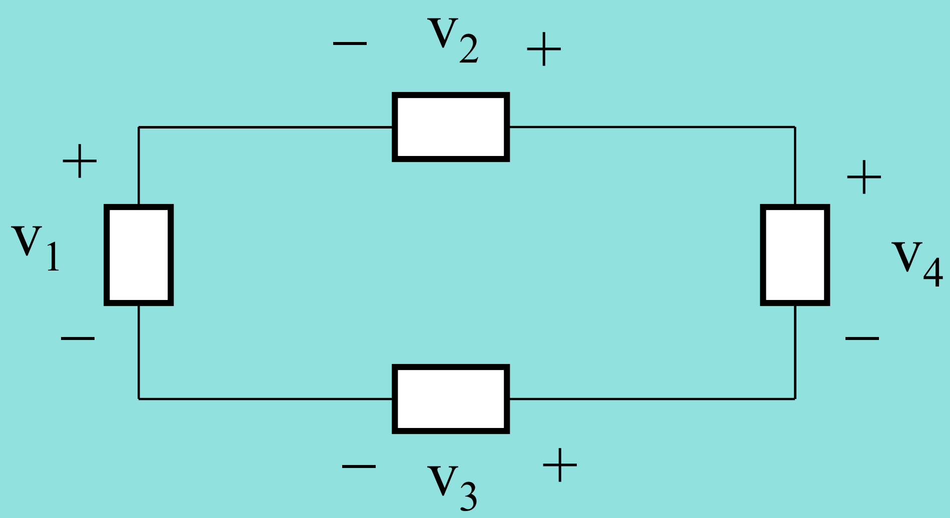


Figure 3.1

Basic Laws of Circuits

<u>Kirchhoff's Voltage Law</u>: **Consideration 1.**

We define a voltage drop as positive if we enter the positive terminal and leave the negative terminal.

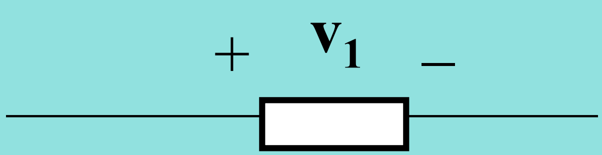


Figure 3.2

The drop moving from left to right above is $+v_1$.

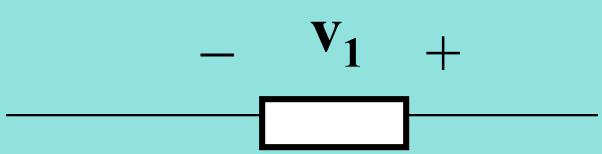


Figure 3.3

The drop moving from left to right above is $-v_1$.

Basic Laws of Circuits

Kirchhoff's Voltage Law: Consider the circuit of Figure 3.4 once again. If we sum the voltage drops in the clockwise direction around the circuit starting at point "a" we write:

$- v_1 - v_2 + v_4 + v_3 = 0$ ♦ **drops in CW direction starting at "a"**

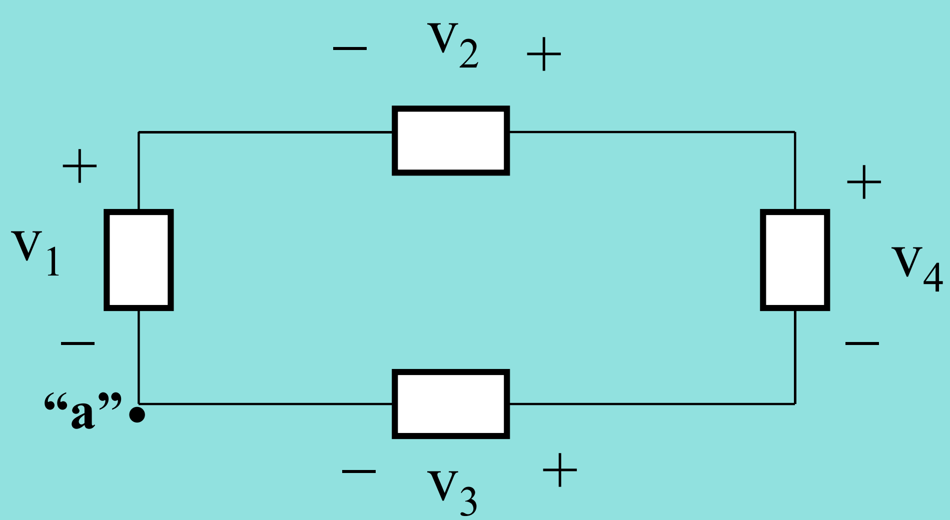


Figure 3.4

$- v_3 - v_4 + v_2 + v_1 = 0$ ♦ **drops in CCW direction starting at "a"**

Basic Laws of Circuits

<u>Kirchhoff's Voltage Law</u>:

<u>Consideration 2</u>: <u>Sum</u> of the voltage <u>rises</u> around a circuit <u>equal zero</u>. We first define a drop.

We define a voltage rise in the following diagrams:

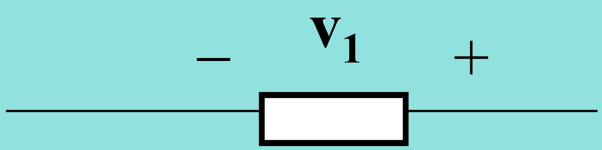


Figure 3.5

The voltage rise in moving from left to right above is $+v_1$.

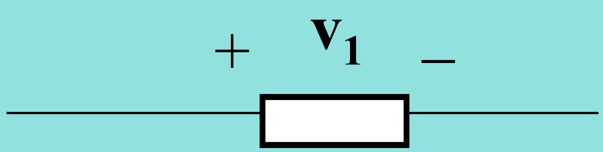


Figure 3.6

The voltage rise in moving from left to right above is $-v_1$.

Basic Laws of Circuits

Kirchhoff's Voltage Law: **Consider the circuit of Figure 3.7 once again. If we sum the voltage rises in the clockwise direction around the circuit starting at point "a" we write:**

$+ v_1 + v_2 - v_4 - v_3 = 0$ **rises in the CW direction starting at "a"**

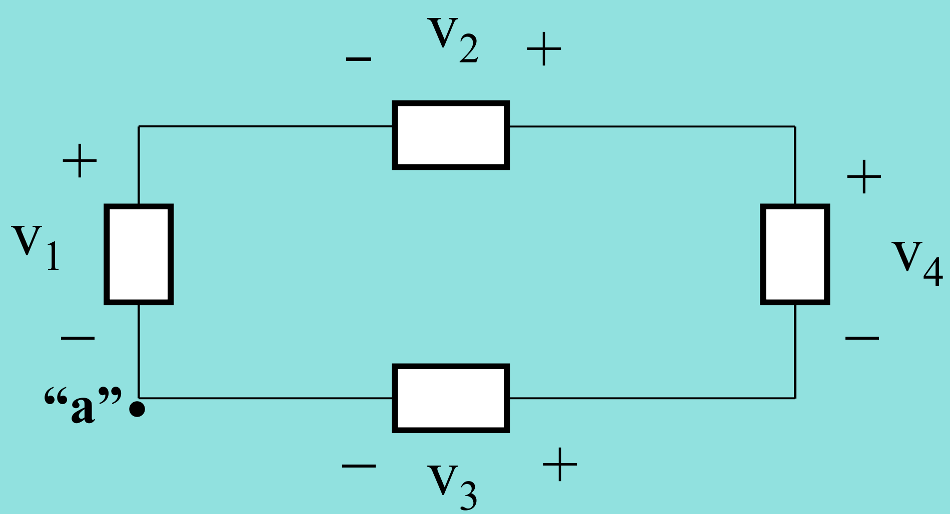


Figure 3.7

$+ v_3 + v_4 - v_2 - v_1 = 0$ **rises in the CCW direction starting at "a"**

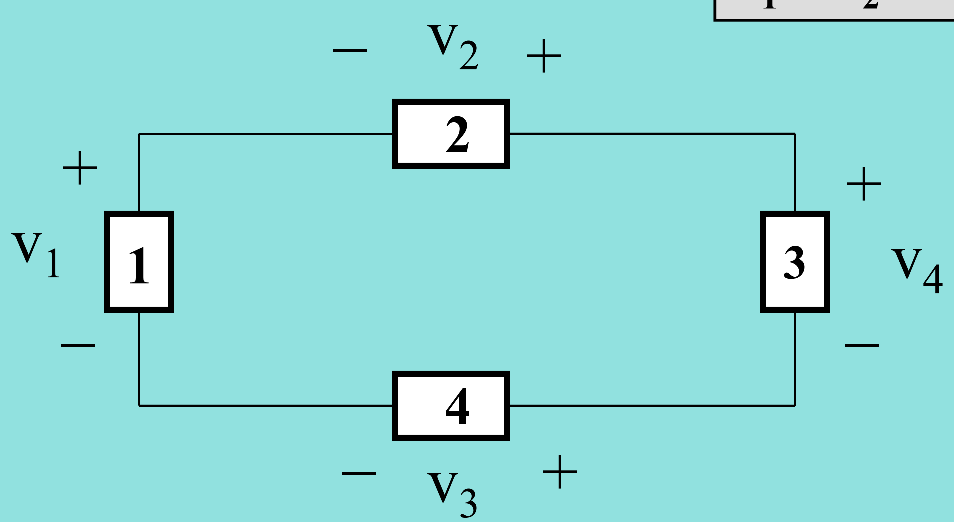
Basic Laws of Circuits

Kirchhoff's Voltage Law:

Consideration 3: **Sum of the voltage rises around a circuit equal the sum of the voltage drops.**

Again consider the circuit of Figure 3.1 in which we start at point "a" and move in the CW direction. As we cross elements 1 & 2 we use voltage rise: as we cross elements 4 & 3 we use voltage drops. This gives the equation,

$$\boxed{v_1 + v_2 = v_4 + v_3}$$



Basic Laws of Circuits

<u>Kirchhoff's Voltage Law</u>: Comments.

- **We note that a positive voltage drop = a negative voltage rise.**
- **We note that a positive voltage rise = a negative voltage drop.**
- **There are similarities in the way we state Kirchhoff's voltage and Kirchhoff's current laws: algebraic sums …**

However, one would *<u>never</u>* say that the sum of the voltages entering a junction point in a circuit equal to zero.

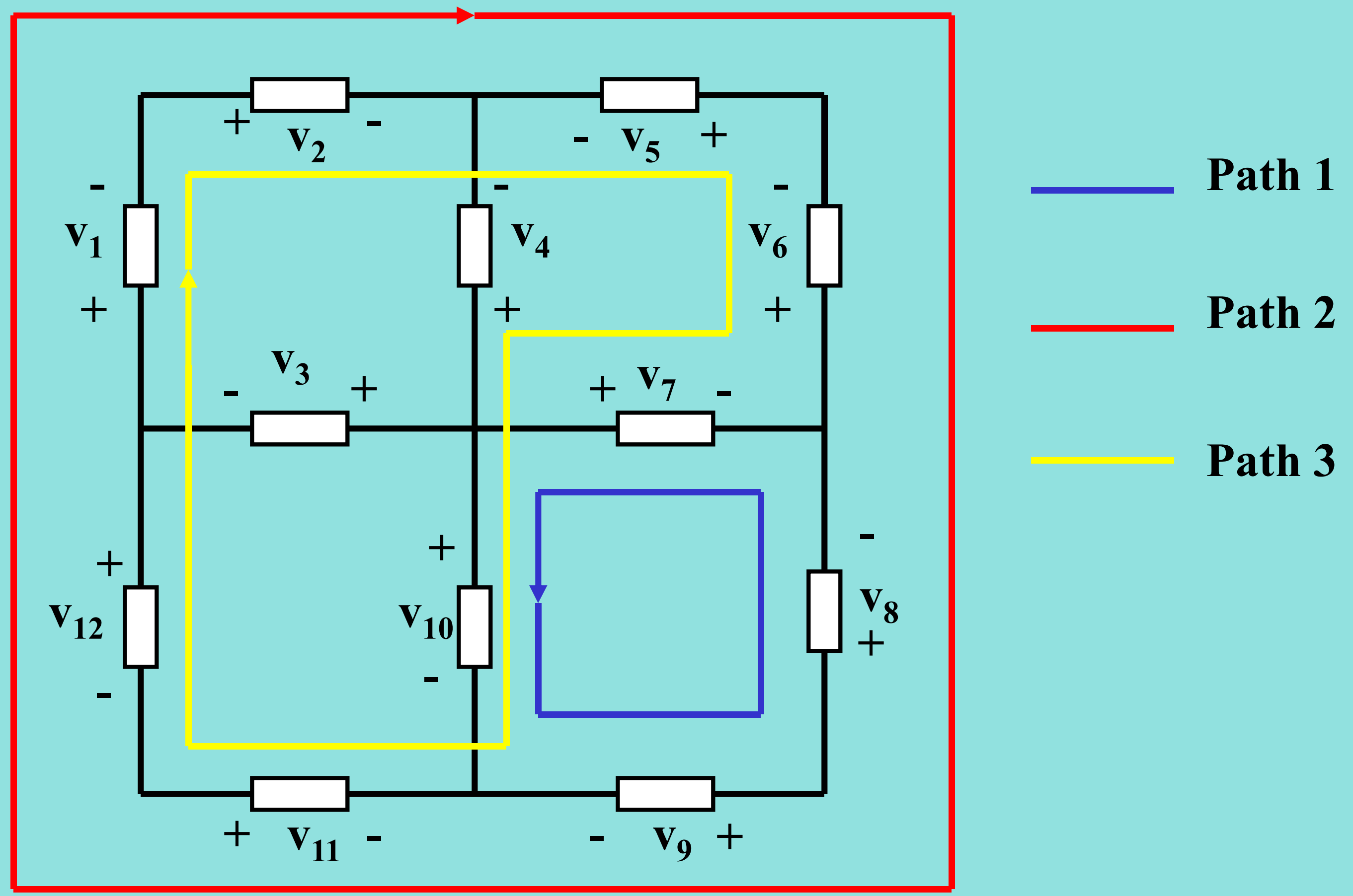
Likewise, one would *never* say that the sum of the currents around a closed path in an electric circuit equal zero.

Basic Laws of Circuits

<u>Kirchhoff's Voltage Law</u>: Further details.

For the circuit of Figure 3.8 there are a number of closed paths. Three have been selected for discussion.

Figure 3.8
Multi-path Circuit.



Basic Laws of Circuits

<u>Kirchhoff's Voltage Law</u>: Further details.

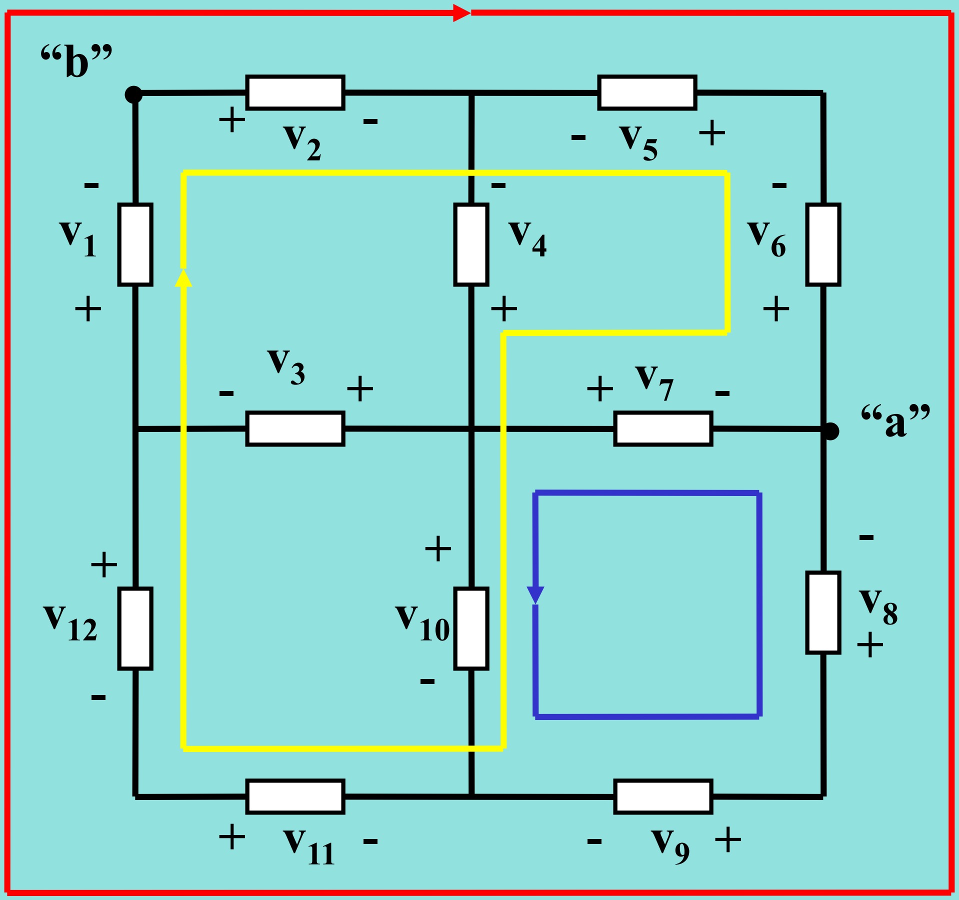
For any given circuit, there are a fixed number of closed paths that can be taken in writing Kirchhoff's voltage law and still have linearly independent equations. We discuss this more, later.

Both the starting point and the direction in which we go around a closed path in a circuit to write Kirchhoff's voltage law are arbitrary. However, one must end the path at the same point from which one started.

Conventionally, in most text, the sum of the voltage drops equal to zero is normally used in applying Kirchhoff's voltage law.

Basic Laws of Circuits

<u>Kirchhoff's Voltage Law</u>: Illustration from Figure 3.8.



Using sum of the drops = 0

Blue path, starting at "a"

$- v_7 + v_{10} - v_9 + v_8 = 0$

Red path, starting at "b"

$+v_2 - v_5 - v_6 - v_8 + v_9 - v_{11} - v_{12} + v_1 = 0$

Yellow path, starting at "b"

$+ v_2 - v_5 - v_6 - v_7 + v_{10} - v_{11} - v_{12} + v_1 = 0$

Basic Laws of Circuits

<u>Kirchhoff's Voltage Law</u>: Double subscript notation.

Voltages in circuits are often described using double subscript notation.

Consider the following:

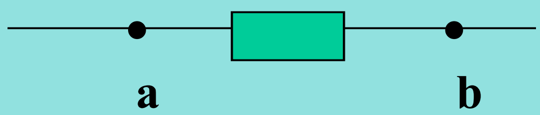


Figure 3.9: Illustrating double subscript notation.

V_{ab} means the potential of point a with respect to point b with point a assumed to be at the highest (+) potential and point b at the lower (-) potential.

Basic Laws of Circuits

Kirchhoff's Voltage Law: Double subscript notation.

Task: Write Kirchhoff's voltage law going in the clockwise direction for the diagram in Figure 3.10.

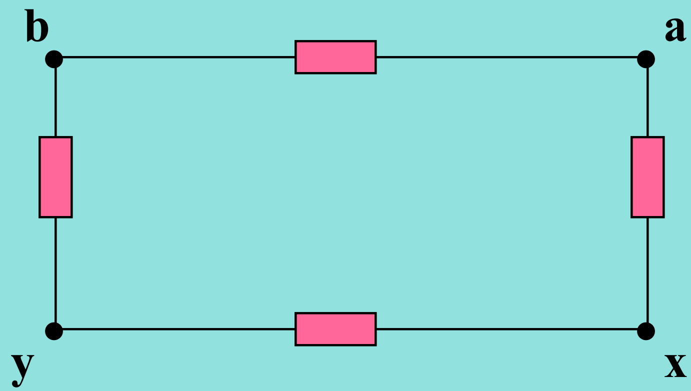


Figure 3.10: Circuit for illustrating double subscript notation.

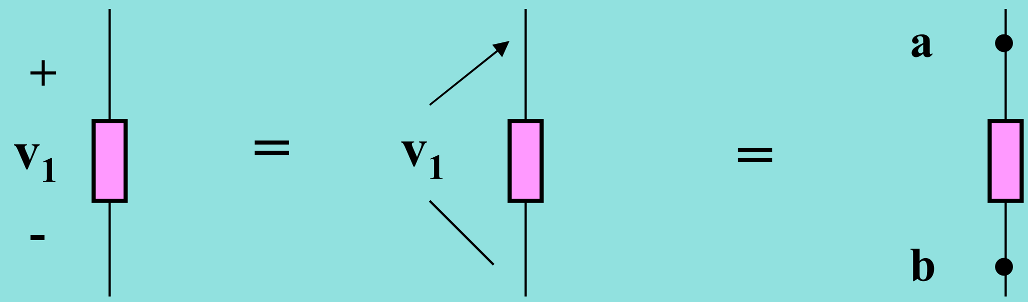
Going in the clockwise direction, starting at "b", using rises;

$$v_{ab} + v_{xa} + v_{yx} + v_{by} = 0$$

Basic Laws of Circuits

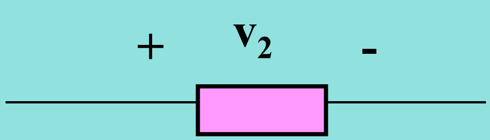
Kirchhoff's Voltage Law: Equivalences in voltage notations

The following are equivalent in denoting polarity.



$v_{ab} = v_1$

Assumes the upper terminal is positive in all 3 cases



$v_2 =$ **- 9 volts means the right hand side of the element is actually positive.**

Basic Laws of Circuits

<u>Kirchhoff's Voltage Law</u>: Application.

Given the circuit of Figure 3.11. Find V_{ad} and V_{fc}.

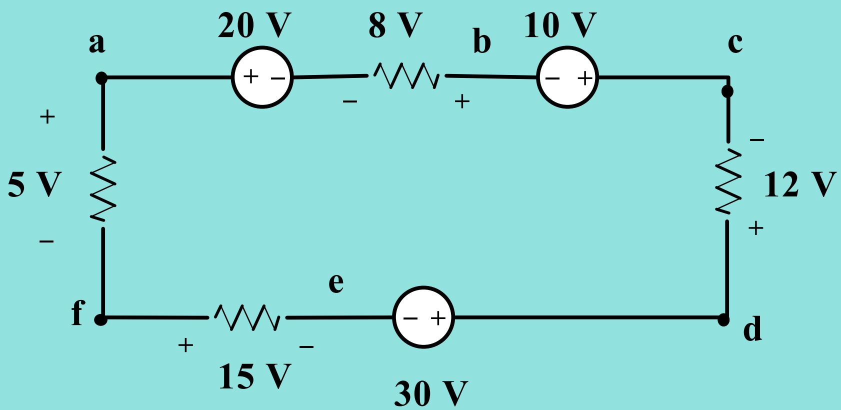


Figure 3.11: Circuit for illustrating KVL.

Using drops = 0; $V_{ad} + 30 - 15 - 5 = 0 \longrightarrow V_{ab} = -10$ V

$V_{fc} - 12 + 30 - 15 = 0 \longrightarrow V_{fc} = -3$ V

Basic Laws of Circuits

Kirchhoff's Voltage Law: Single-loop circuits.

We are now in a position to combine Kirchhoff's voltage and current Laws to the solution of single loop circuits. We start by developing the *Voltage Divider Rule.* Consider the circuit of Figure 3.12.

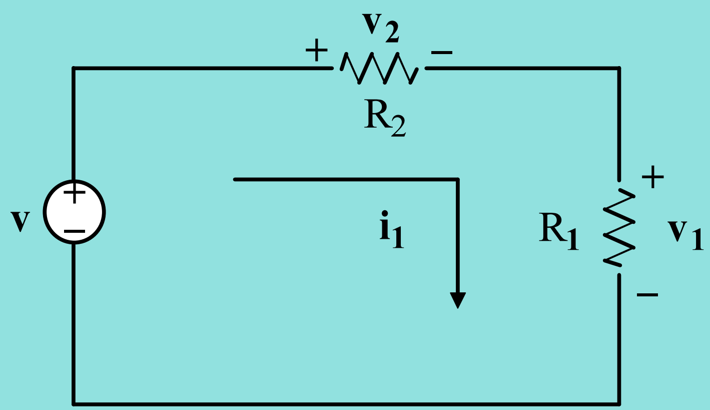


Figure 3.12: Circuit for developing voltage divider rule.

$$v = v_1 + v_2$$

$$v_1 = i_1 R_1, \quad v_2 = i_1 R_2$$

then,

$$v = i_1(R_1 + R_2) \text{ , and } \quad i_1 = \frac{v}{(R_1 + R_2)}$$

so,

$$\boxed{v_1 = \frac{vR_1}{(R_1 + R_2)} \;\; *}$$

* You will be surprised by how much you use this in circuits.

Basic Laws of Circuits

<u>Kirchhoff's Voltage Law</u>: Single-loop circuits.

Find V_1 in the circuit shown in Figure 3.13.

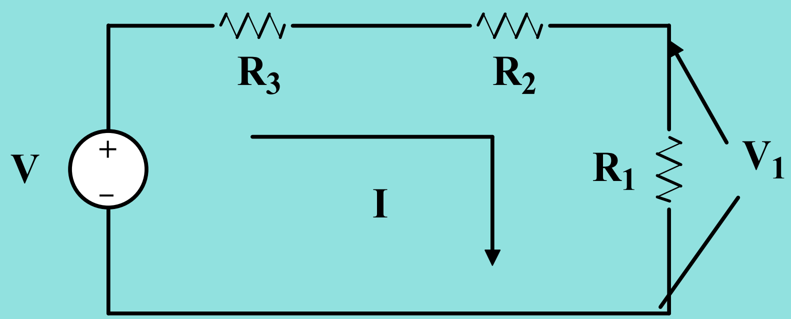


Figure 3.13

$$I = \frac{V}{(R_1 + R_2 + R_3)}$$

$$V_1 = IR_1 \quad ,so, we have$$

$$V_1 = \frac{VR_1}{(R_1 + R_2 + R_3)}$$

Basic Laws of Circuits

Kirchhoff's Voltage Law: Single-loop circuits.

Example 3.1: For the circuit of Figure 3.14, the following is known:

R_1 = 4 ohms, R_2 = 11 ohms, V = 50 volts, P_1 = 16 watts

Find R_3.

Solution:

P_1 = 16 watts = I^2R_1, thus,

I = 2 amps

$V = I(R_1 + R_2 + R_3)$, giving,

$R_1 + R_2 + R_3 = 25$, then solve for R_3,

$R_3 = 25 – 15 = 10$ ohms

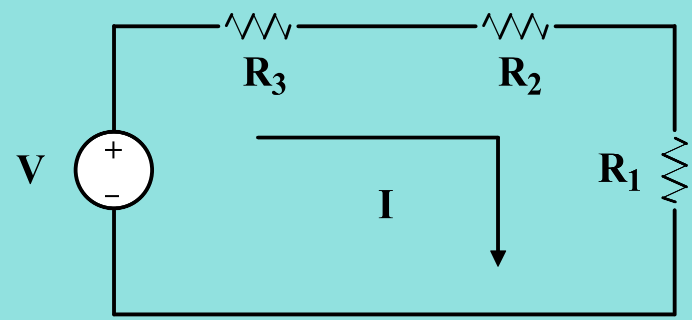


Figure 3.14: Circuit for example 3.1.

Basic Laws of Circuits

<u>Kirchhoff's Voltage Law</u>: Single-loop circuits.

Example 3.2: For the circuit in Figure 3.15 find I, V_1, V_2, V_3, V_4 and the power supplied by the 10 volt source.

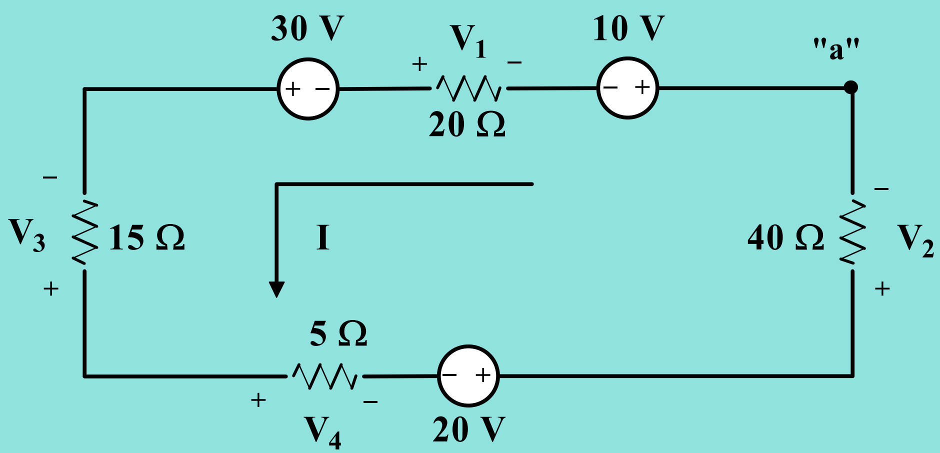


Figure 3.15: Circuit for example 3.2.

For convenience, we start at point "a" and sum voltage drops =0 in the direction of the current I.

$$+10 - V_1 - 30 - V_3 + V_4 - 20 + V_2 = 0 \qquad \text{Eq. 3.1}$$

Basic Laws of Circuits

Kirchhoff's Voltage Law: Single-loop circuits. Ex. 3.2 cont.

We note that: $V_1 = -20I$, $V_2 = 40I$, $V_3 = -15I$, $V_4 = 5I$ Eq. 3.2

We substitute the above into Eq. 3.1 to obtain Eq. 3.3 below.

$$10 + 20I - 30 + 15I + 5I - 20 + 40I = 0 \qquad \text{Eq. 3.3}$$

Solving this equation gives, I = 0.5 A.

Using this value of I in Eq. 3.2 gives;

$V_1 = -10$ V $\qquad V_3 = -7.5$ V

$V_2 = 20$ V $\qquad V_4 = 2.5$ V

$$P_{10(supplied)} = -10I = -5 \text{ W}$$

(We use the minus sign in –10I because the current is entering the + terminal)
In this case, power is being absorbed by the 10 volt supply.

Basic Laws of Circuits

Kirchhoff's Voltage Law: Single-loop circuits, Equivalent Resistance.

Given the circuit of Figure 3.16. We desire to develop an equivalent circuit as shown in Figure 3.17. Find V_s and R_{eq}.

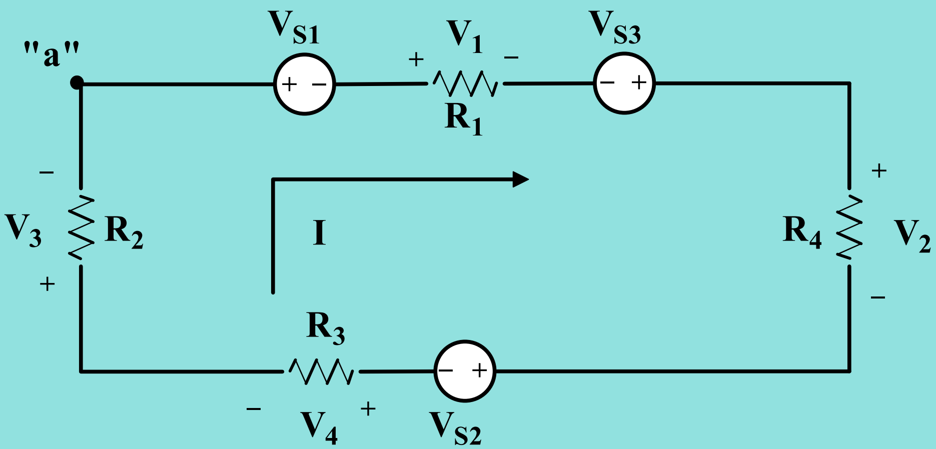


Figure 3.16: Initial circuit for development.

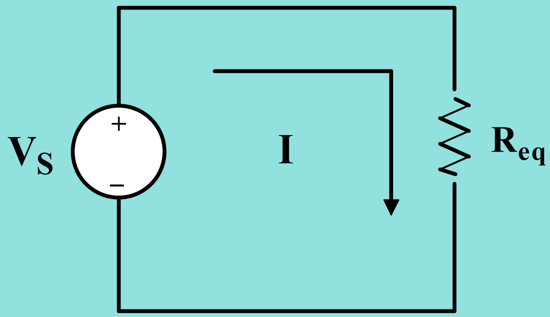


Figure 3.17: Equivalent circuit for Figure 3.16

Basic Laws of Circuits

<u>Kirchhoff's Voltage Law</u>: Single-loop circuits, Equivalent Resistance.

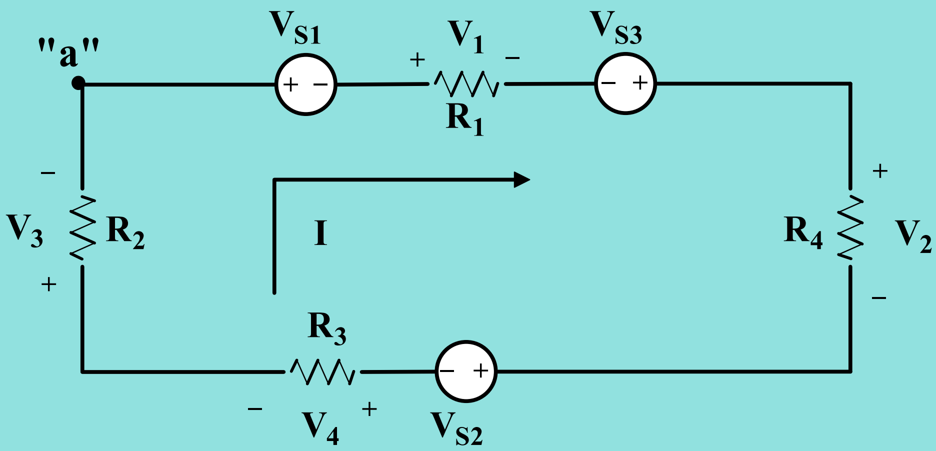


Figure 3.16: Initial circuit.

Starting at point "a", apply KVL going clockwise, using drops = 0, we have

$$V_{S1} + V_1 - V_{S3} + V_2 + V_{S2} + V_4 + V_3 = 0$$

or

$$-V_{S1} - V_{S2} + V_{S3} = I(R_1 + R_2 + R_3 + R_4) \qquad \text{Eq. 3.4}$$

Basic Laws of Circuits

<u>Kirchhoff's Voltage Law</u>: Single-loop circuits, Equivalent Resistance.

Consider again, the circuit of Figure 3.17.

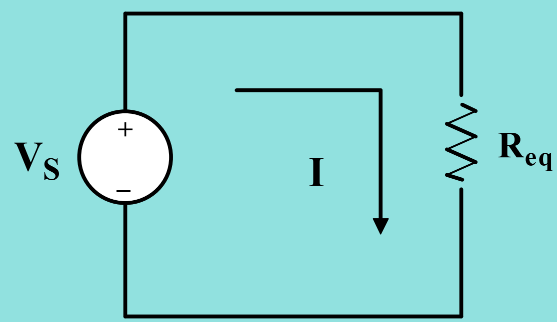


Figure 3.17: Equivalent circuit of Figure 3.16.

Writing KVL for this circuit gives;

$V_S = IR_{eq}$ **compared to** $-V_{S1} - V_{S2} + V_{S3} = I(R_1 + R_2 + R_3 + R_4)$

Therefore;

$$\boxed{V_S = -V_{S1} - V_{S2} + V_{S3}\ ;} \qquad \boxed{R_{eq} = R_1 + R_2 + R_3 + R_4} \qquad \textbf{Eq. 3.5}$$

Basic Laws of Circuits

<u>Kirchhoff's Voltage Law</u>: Single-loop circuits, Equivalent Resistance.

We make the following important observations from Eq. 3.5:

- **The equivalent source of a single loop circuit can be obtained by summing the rises around the loop of the individual sources.**

- **The equivalent resistance of resistors in series is equal to the sum of the individual resistors.**

Basic Laws of Circuits

<u>Kirchhoff's Voltage Law</u>: Single-loop circuits.

Example 3.3: Find the current I in the circuit of Figure 3.18.

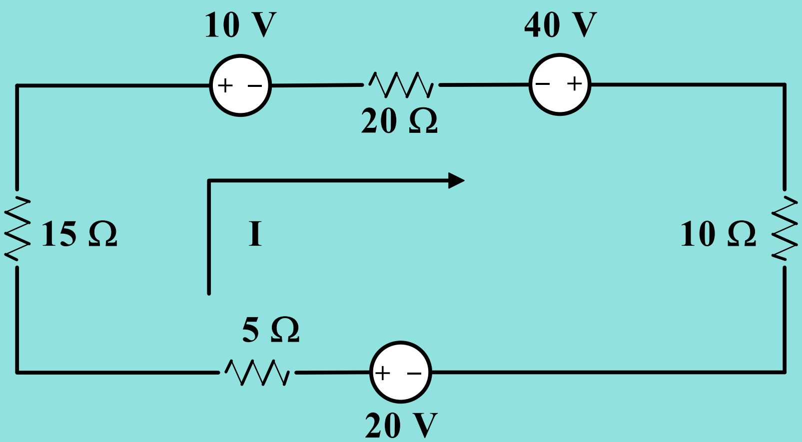
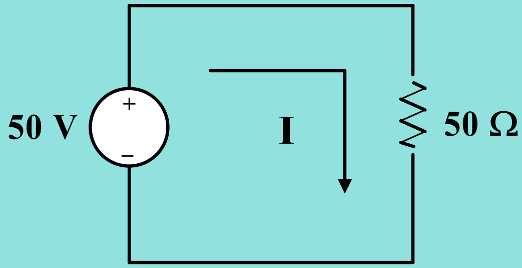


Figure 3.18: Circuit for example 3.3.

From the previous discussion we have the following circuit.



Therefore, I = 1 A