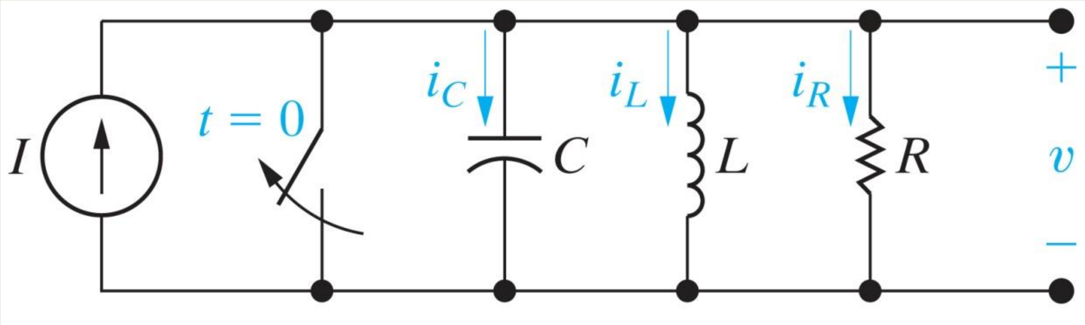
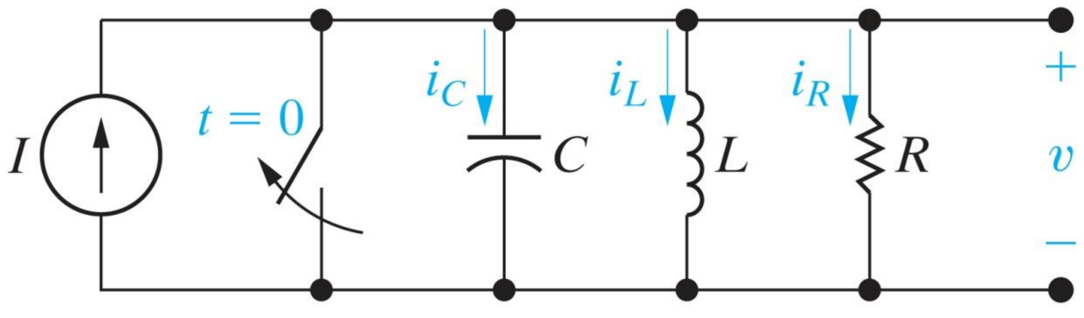
Step Response of Second-order RLC Circuits

The problem – find the response for $t \geq 0$. Note that there may or may not be initial energy stored in the inductor and capacitor!



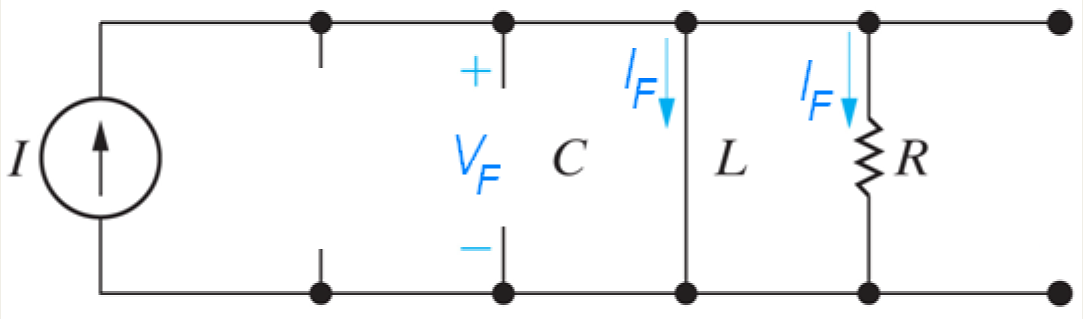
The circuit for the parallel RLC step response is repeated here. Consider how this circuit behaves as $t \rightarrow \infty$. Which component's final value is non-zero?



A. The resistor
B. The inductor
C. The capacitor
D. All of the above

Step Response of a Parallel RLC Circuits

As $t \to \infty$:

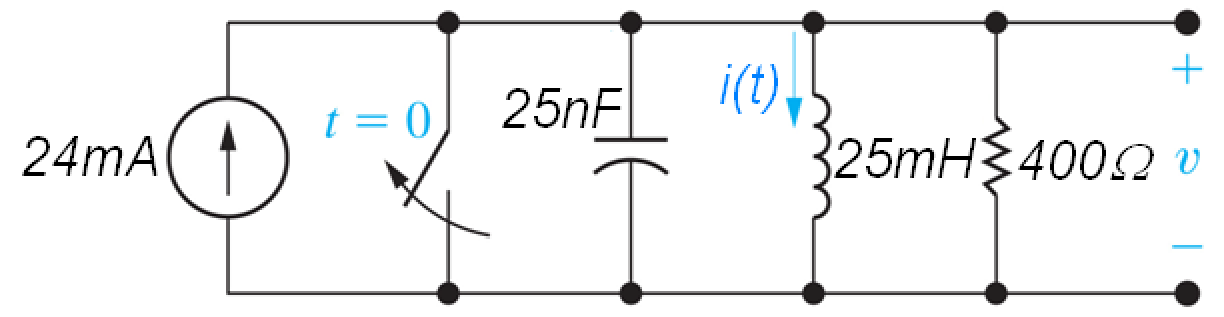


The only component whose final value is NOT zero is the inductor, whose final current is the current supplied by the source. We now have to construct a response form that can satisfy two initial conditions and one non-zero final value. We can satisfy the final value directly if we specify the inductor current as the response we will solve for:

$$i_L(t) = I_F + (\text{the form of the natural response})$$

Step Response of a Parallel RLC Circuit

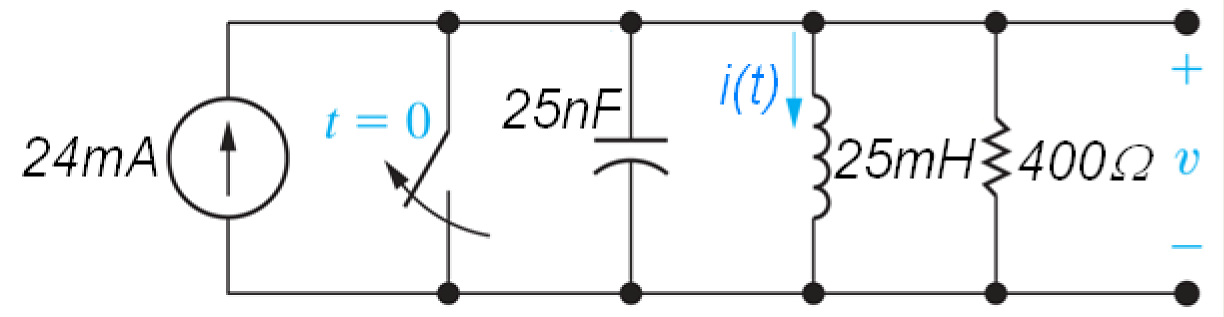
The problem – there is no initial energy stored in this circuit; find *i(t)* for $t \geq 0$.



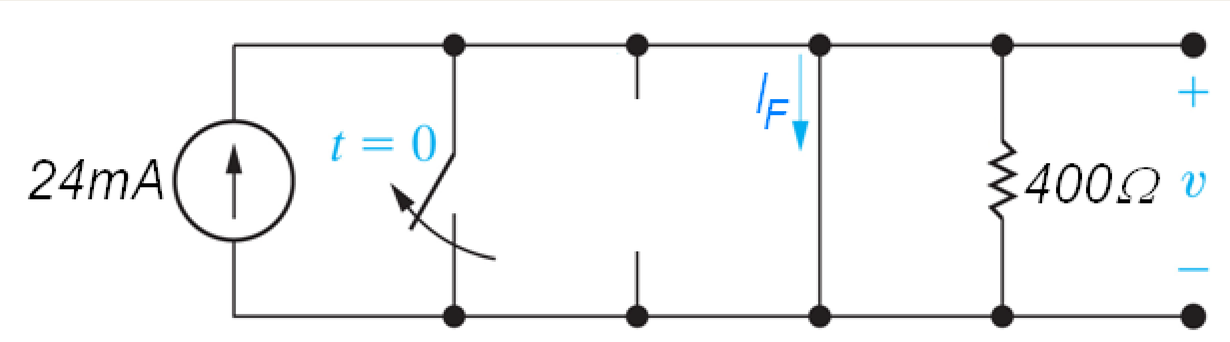
To begin, find the initial conditions and the final value. The initial conditions for this problem are both zero; the final value is found by analyzing the circuit as $t \to \infty$.

Step Response of a Parallel RLC Circuit

The problem – there is no initial energy stored in this circuit; find *i(t)* for $t \geq 0$.



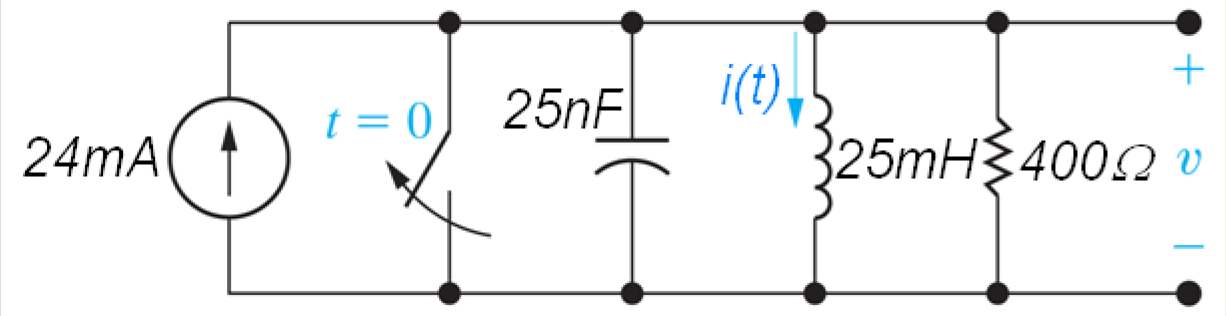
$t \to \infty$:



$$I_F = 24\text{mA}$$

Step Response of a Parallel RLC Circuit

The problem – there is no initial energy stored in this circuit; find *i(t)* for *t ≥ 0*.



Next, calculate the values of α and ω_0 and determine the form of the response:

$$\alpha = \frac{1}{2RC} = \frac{1}{2(400)(25\text{n})} = 50{,}000\text{rad/s}$$

$$\omega_0 = \sqrt{1/LC} = \sqrt{1/(25\text{m})(25\text{n})} = 40{,}000\text{rad/s}$$

We just calculated $\alpha = 50{,}000$ rad/s and $\omega_0 = 40{,}000$ rad/s, so the form of the response is

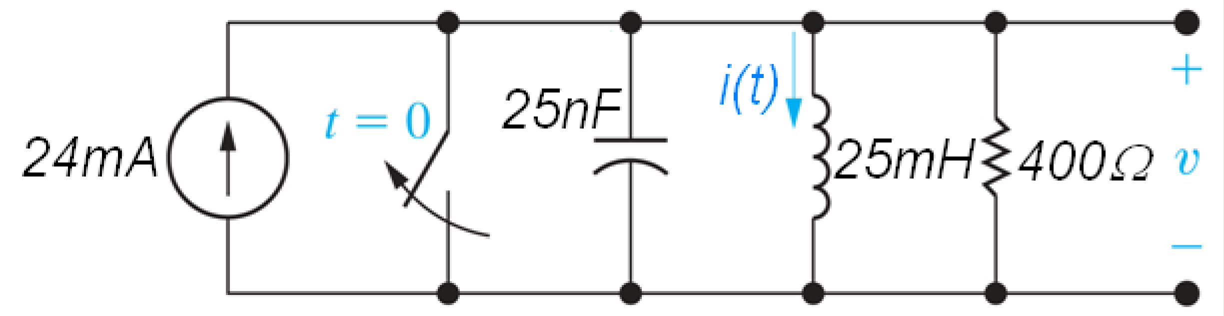
A. Overdamped
B. Underdamped
C. Critically damped

Once we know the response form is overdamped, we know we have to calculate

A. ω_d
B. s_1 and s_2
C. Nothing additional

Step Response of a Parallel RLC Circuit

The problem – there is no initial energy stored in this circuit; find *i(t)* for *t ≥ 0*.



Since the response form is overdamped, calculate the values of s_1 and s_2:

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -50{,}000 \pm \sqrt{50{,}000^2 - 40{,}000^2}$$

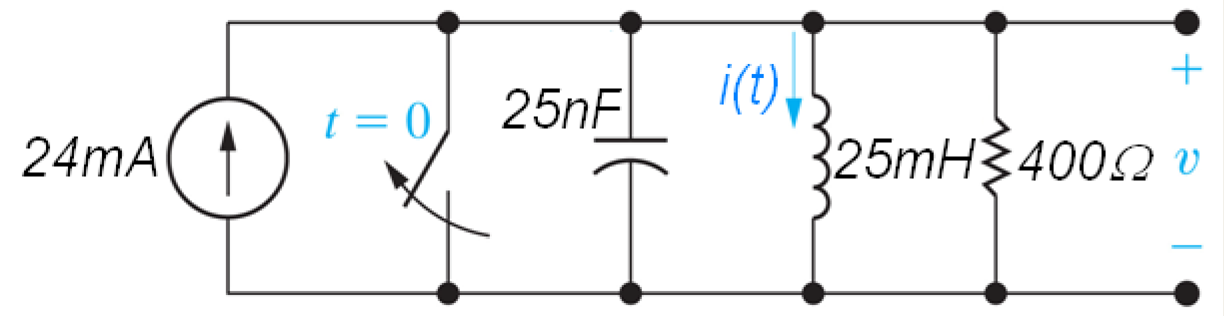
$$= -50{,}000 \pm 30{,}000 \text{ rad/s}$$

$$\therefore \qquad s_1 = -20{,}000 \text{ rad/s} \text{ and } s_2 = -80{,}000 \text{ rad/s}$$

$$\Rightarrow \qquad i_L(t) = 0.024 + A_1 e^{-20{,}000t} + A_2 e^{-80{,}000t} \text{ A}, t \geq 0$$

Step Response of a Parallel RLC Circuit

The problem – there is no initial energy stored in this circuit; find *i(t)* for *t ≥ 0*.



$$i_L(t) = 0.024 + A_1 e^{-20{,}000t} + A_2 e^{-80{,}000t}\ \text{A}, t \geq 0$$

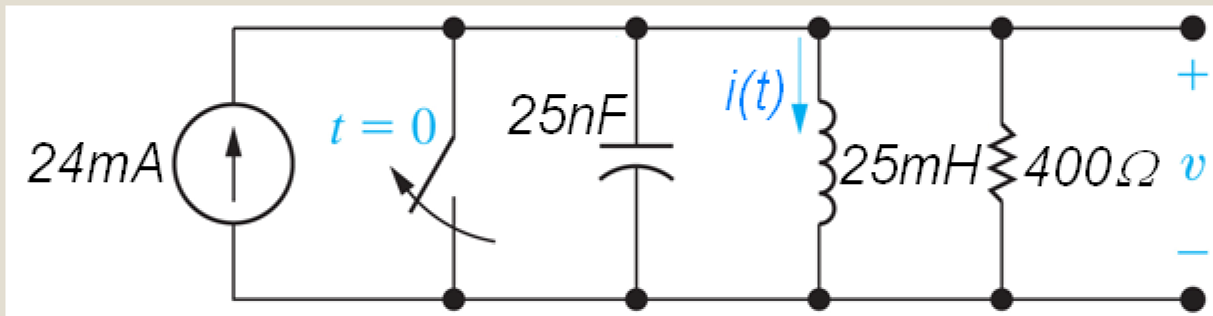
Next, set the values of *i(0)* and *di(0)/dt* from the equation equal to the values of *i(0)* and di(0)/dt from the circuit.

From the equation: $i_L(0) = 0.024 + A_1 + A_2$

From the circuit: $i_L(0) = I_0 = 0$

Step Response of a Parallel RLC Circuit

The problem – there is no initial energy stored in this circuit; find *i(t)* for *t ≥ 0*.



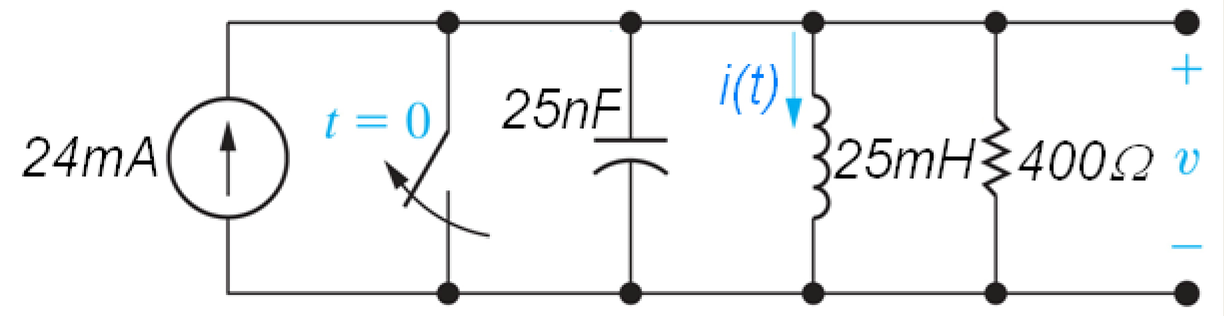
$$i_L(t) = 0.024 + A_1 e^{-20{,}000t} + A_2 e^{-80{,}000t} \text{ A}, t \geq 0$$

From the equation: $\dfrac{di_L(0)}{dt} = -20{,}000A_1 - 80{,}000A_2$

From the circuit: $\dfrac{di_L(0)}{dt} = \dfrac{v_L(0)}{L} = \dfrac{V_0}{L} = 0$

Step Response of a Parallel RLC Circuit

The problem – there is no initial energy stored in this circuit; find *i(t)* for $t \geq 0$.



$$i_L(t) = 0.024 + A_1 e^{-20{,}000t} + A_2 e^{-80{,}000t} \text{ A}, t \geq 0$$

$$\text{Solve:} \quad 0.024 + A_1 + A_2 = 0$$

$$\text{and} \quad -20{,}000 A_1 - 80{,}000 A_2 = 0$$

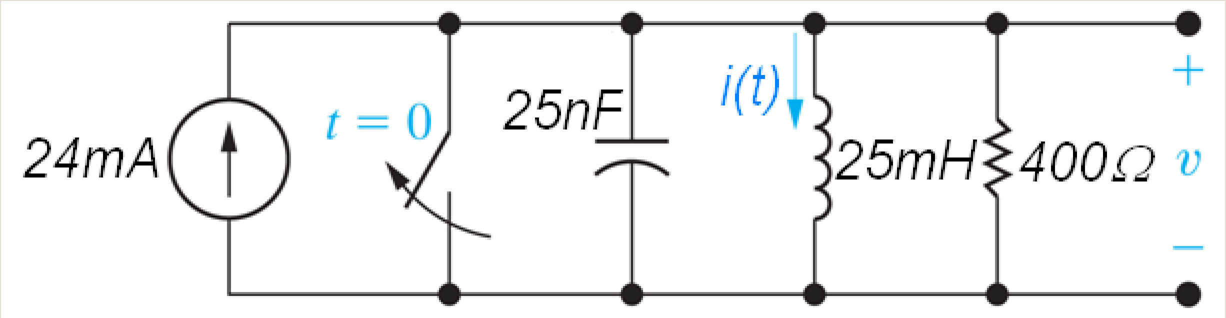
$$\therefore \quad A_1 = -32\,\text{mA}; \quad A_2 = 8\,\text{mA}$$

$$\Rightarrow \quad i_L(t) = 24 - 32e^{-20{,}000t} + 8e^{-80{,}000t} \text{ mA}, t \geq 0$$

$$i_L(t) = 24 - 32e^{-20,000t} + 8e^{-80,000t} \text{ mA}, t \geq 0$$

We can check this result at $t = 0$ and as $t \to \infty$; from the equation we get

- A. $i_L(0)=32$ mA, $i_L(\infty)=0$
- B. $i_L(0)=0$, $i_L(\infty)=0$
- C. $i_L(0)=0$, $i_L(\infty)=24$ mA



$$i_L(t) = 24 - 32e^{-20{,}000t} + 8e^{-80{,}000t}\ \text{mA}, t \geq 0$$

If we now want to find *v(t)* for *t ≥ 0*, we need to

A. Find the derivative of the current and multiply by L
B. Find the integral of the current and divide by C
C. Multiply the current by R

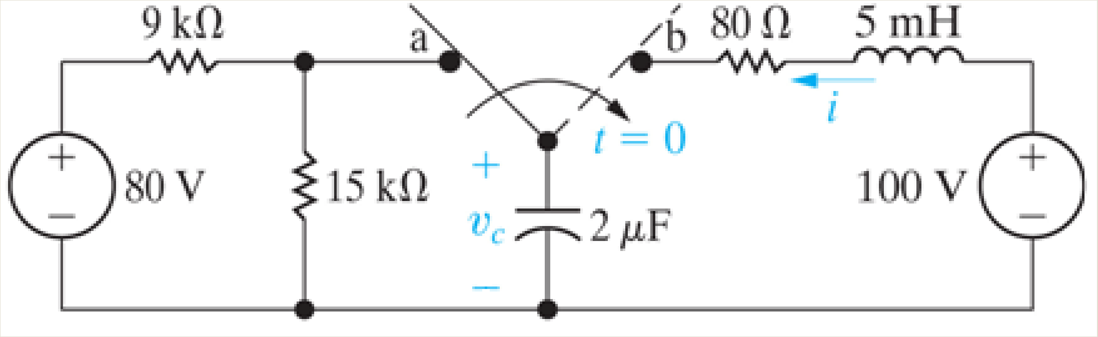
Step Response of RLC Circuits – Summary

Use the Second-Order Circuits table:

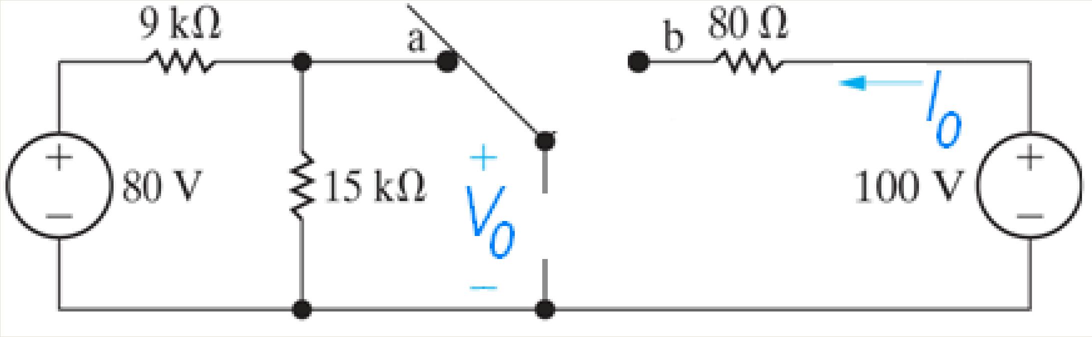
1. Make sure you are on the Step Response side.
2. Find the appropriate column for the RLC circuit topology.
3. Find the values of the two initial conditions and the one non-zero final value.Make sure the initial conditions and final value are defined exactly as shown in the figure!
4. Use the equations in Row 4 to calculate α and ω_0.
5. Compare the values of α and ω_o to determine the response form (given in one of the last 3 rows).
6. Write the equation for $v_C(t)$, $t \geq 0$ (series) or $i_L(t)$, $t \geq 0$ (parallel), leaving only the 2 coefficients unspecified.
7. Use the equations provided to solve for the unknown coefficients.
8. Solve for any other quantities requested in the problem.

Step Response of a Series RLC Circuit

The problem –
find $v_C(t)$ for $t \geq 0$.



Find the initial conditions by analyzing the circuit for $t < 0$:

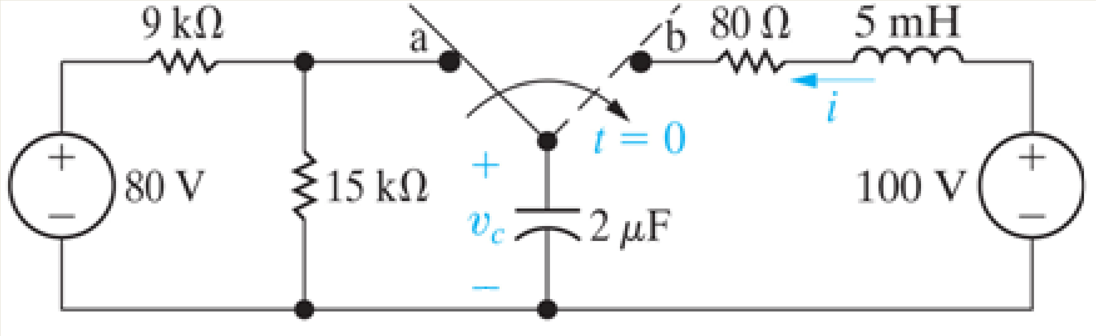


$$V_0 = \frac{15\text{k}}{9\text{k} + 15\text{k}}(80\,\text{V})$$
$$= 50\,\text{V}$$

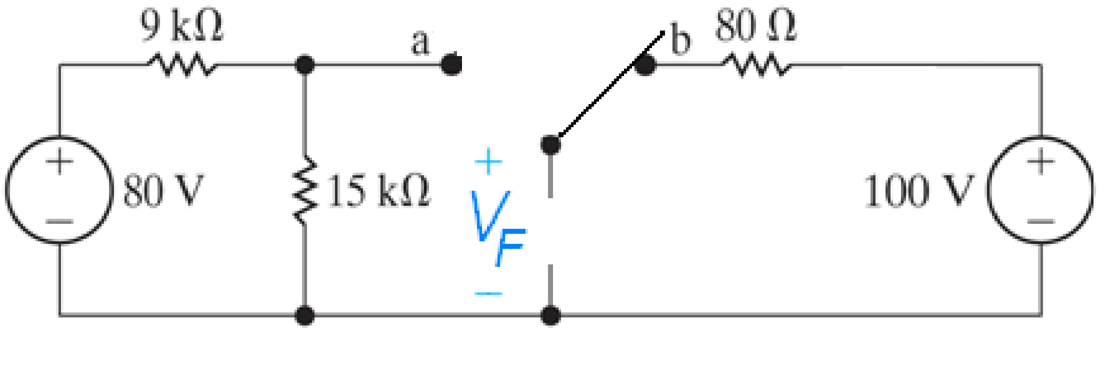
$$I_0 = 0\,\text{A}$$

Step Response of a Series RLC Circuit

The problem – find $v_C(t)$ for $t \geq 0$.



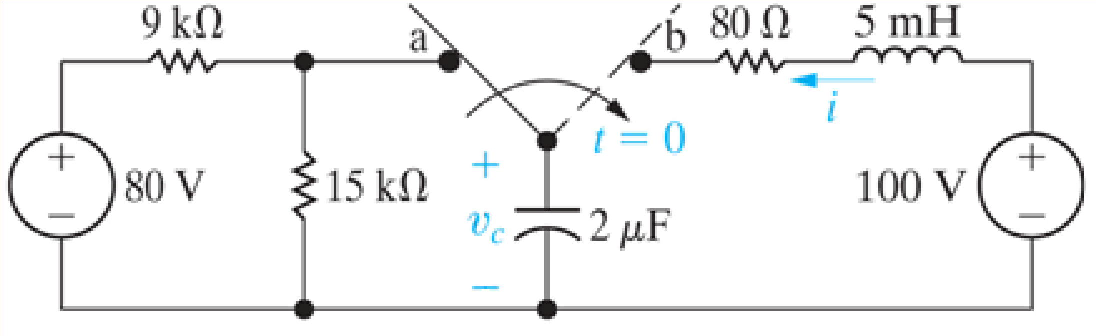
Find the final value of the capacitor voltage by analyzing the circuit as $t \to \infty$:



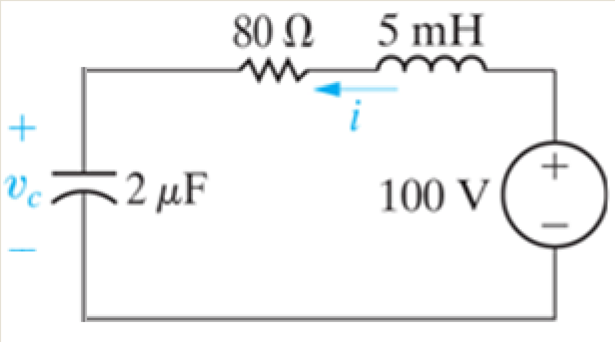
$$V_F = 100\text{V}$$

Step Response of a Series RLC Circuit

The problem –
find $v_C(t)$ for $t \geq 0$.



Use the circuit for $t \geq 0$ to find the values of α and ω_0:



$$\alpha = R/2L = 80/2(0.005) = 8000\text{rad/s}$$

$$\omega_0 = \sqrt{1/LC} = \sqrt{1/(0.005)(2\mu)}$$

$$= 10{,}000\text{rad/s}$$

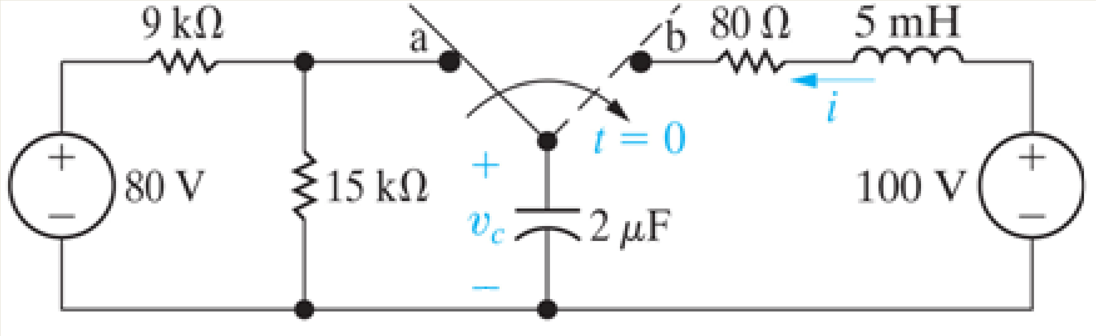
$$\alpha^2 < \omega_0^2 \qquad \Rightarrow \qquad \text{underdamped}$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{10{,}000^2 - 8000^2}$$

$$= 6000\text{rad/s}$$

Step Response of a Series RLC Circuit

The problem –
find $v_C(t)$ for $t \geq 0$.



Write the equation for the response and solve for the unknown coefficients:

$$v_C(t) = 100 + B_1 e^{-8000t} \cos 6000t + B_2 e^{-8000t} \sin 6000t \text{ V}, t \geq 0$$

$$v_C(0) = V_F + B_1 = V_0 \qquad \therefore \qquad 100 + B_1 = 50$$

$$\frac{dv_C(0)}{dt} = -\alpha B_1 + \omega_d B_2 = \frac{I_0}{C} \qquad \therefore \qquad -8000B_1 + 6000B_2 = 0$$

$$\Rightarrow \qquad B_1 = -50 \text{ V}, \qquad B_2 = 66.67 \text{ V}$$

$$v_C(t) = 100 - 50e^{-8000t} \cos 6000t + 66.67e^{-8000t} \sin 6000t \text{ V}, t \geq 0$$