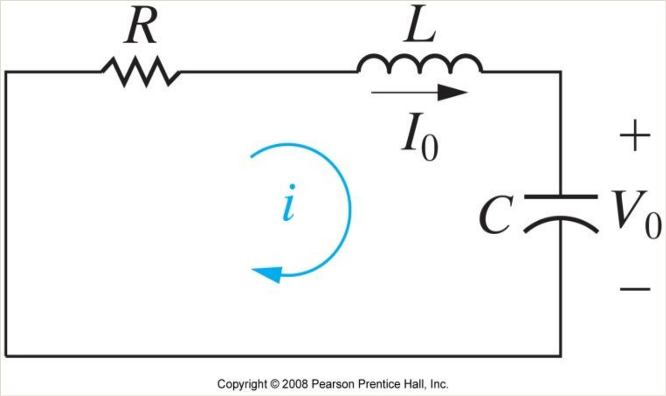
Natural Response of Series RLC Circuits

The problem – given initial energy stored in the inductor and/or capacitor, find *i(t)* for $t \geq 0$.



The difference(s) between the analysis of series RLC circuit and the parallel RLC circuit is/are:

- ✗ A. The variable we calculate.
- ✗ B. The describing differential equation.
- ✗ C. The equations for satisfying the initial conditions
- ✓ D. All of the above

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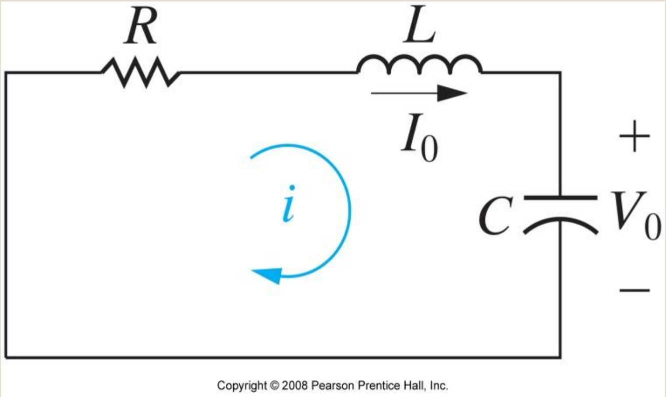


KVL : $$L\frac{di(t)}{dt} + \frac{1}{C}\int_0^t i(x)dx + V_0 + Ri(t) = 0$$

Differentiate both sides to remove the integral : $$L\frac{d^2 i(t)}{dt^2} + \frac{1}{C}i(t) + R\frac{di(t)}{dt} = 0$$

Divide both sides by L to place in standard form : $$\frac{d^2 i(t)}{dt^2} + \frac{R}{L}\frac{di(t)}{dt} + \frac{1}{LC}i(t) = 0$$

The describing differential equation for the series RLC circuit is

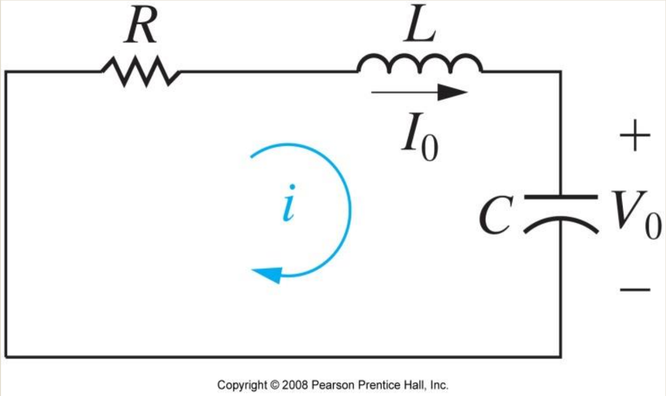
$$\frac{d^2 i(t)}{dt^2} + \frac{R}{L}\frac{di(t)}{dt} + \frac{1}{LC} i(t) = 0$$

Therefore, the characteristic equation is

- ✗ A. $s^2 + (1/RC)s + 1/LC = 0$
- ✓ B. $s^2 + (R/L)s + 1/LC = 0$
- ✗ C. $s^2 + (1/LC)s + 1/RC = 0$

Natural Response of Series RLC Circuits

The problem – given initial energy stored in the inductor and/or capacitor, find *i(t)* for *t ≥ 0*.



The two solutions to the characteristic equation can be calculated using the quadratic formula:

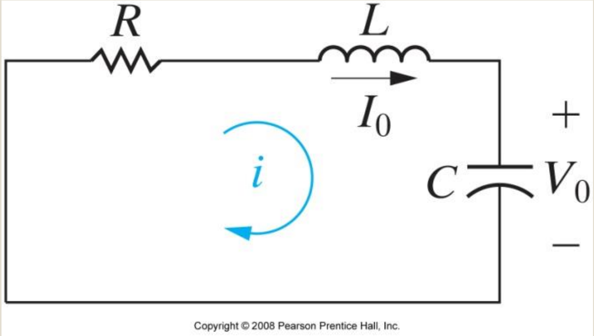
$$s^2 + (R/L)s + (1/LC) = 0; \qquad s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

where $\alpha = \dfrac{R}{2L}$ (the neper frequency in rad/s)

and $\omega_0 = \sqrt{\dfrac{1}{LC}}$ (the resonant radian frequency in rad/s)

Natural Response Series RLC Problems

The problem – given initial energy stored in the inductor and/or capacitor, find *i(t)* for *t ≥ 0*.

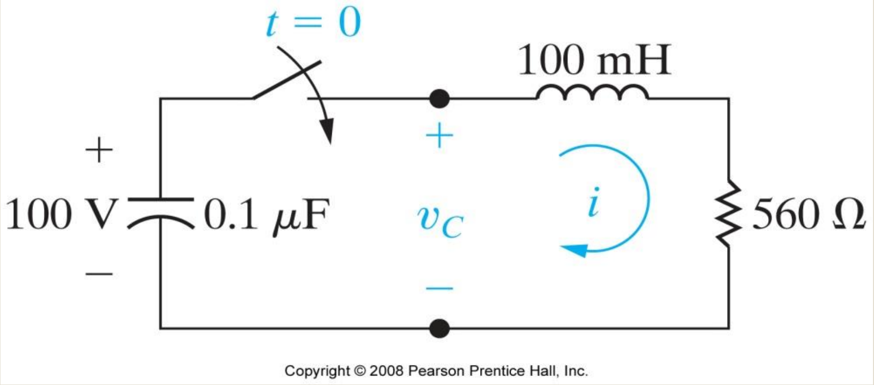


You can solve these problem using the Second-Order Circuits table:

1. Make sure you are on the Natural Response side.
2. Find the series RLC column.
3. Use the equations in Row 4 to calculate α and ω_0.
4. Compare the values of α and ω_o to determine the response form (given in one of the last 3 rows).
5. Use the equations to solve for the unknown coefficients.
6. Write the equation for *i(t), t ≥ 0*.
7. Solve for any other quantities requested in the problem.

Natural Response Series RLC Example

The capacitor is charged to 100 V and at t = 0, the switch closes. Find i(t) for t ≥ 0.



$$\alpha = \frac{R}{2L} = \frac{560}{2(0.1)} = 2800 \text{rad/s}$$

$$\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(0.1)(0.1\mu)}} = 10{,}000 \text{rad/s}$$

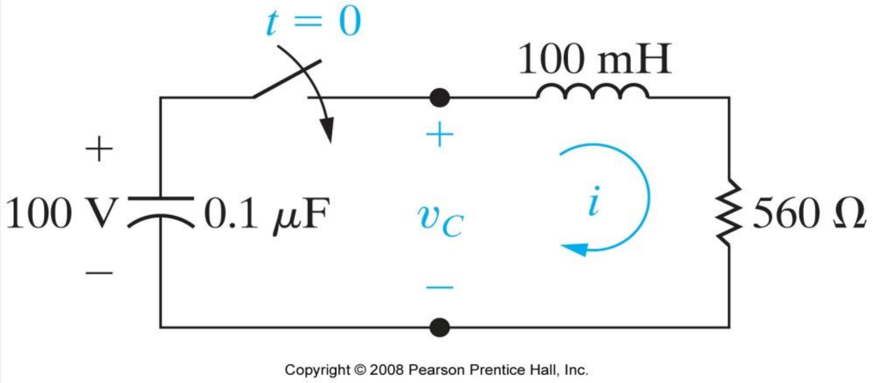
$\alpha^2 < \omega_o^2$ so this is the underdamped case!

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 9600 \text{rad/s}$$

$$i(t) = B_1 e^{-2800t} \cos 9600t + B_2 e^{-2800t} \sin 9600t \text{ A}, t \geq 0$$

Natural Response Series RLC Example

The capacitor is charged to 100 V and at t = 0, the switch closes. Find i(t) for t ≥ 0.



$$i(t) = B_1 e^{-2800t}\cos 9600t + B_2 e^{-2800t}\sin 9600t \text{ A}, t \geq 0$$

Now we must use the coefficients in the equation to satisfy the initial conditions in the circuit:

$$\left. i(t) \right|_{t=0} \text{ in the equation} = \left. i(t) \right|_{t=0} \text{ in the circuit}$$

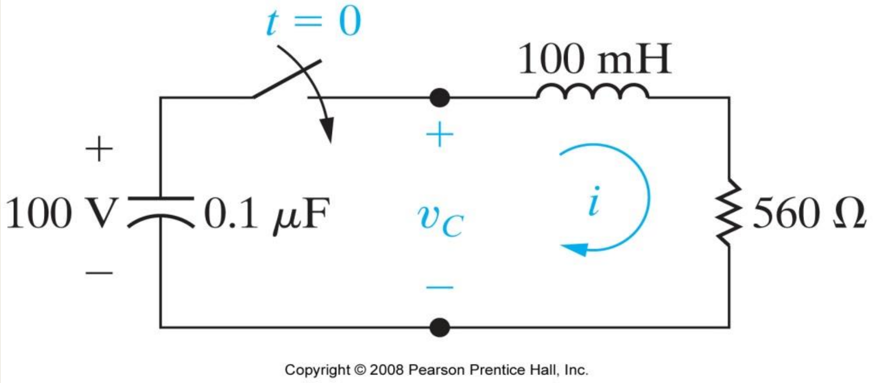
$$\left. \frac{di(t)}{dt} \right|_{t=0} \text{ in the equation} = \left. \frac{di(t)}{dt} \right|_{t=0} \text{ in the circuit}$$

The following quantities used to calculate the unknown coefficients are defined by different equations in both the series and parallel RLC natural response problems:

- ✗ A. The initial values of voltage or current from the equation.
- ✗ B. The initial values of voltage or current from the circuit.
- ✗ C. The initial values of the derivative of voltage or current from the equation.
- ✓ D. The initial values of the derivative of voltage or current from the circuit.

Natural Response Series RLC Example

The capacitor is charged to 100 V and at t = 0, the switch closes. Find i(t) for t ≥ 0.



$$i(t) = B_1 e^{-2800t}\cos 9600t + B_2 e^{-2800t}\sin 9600t \text{ A}, t \geq 0$$

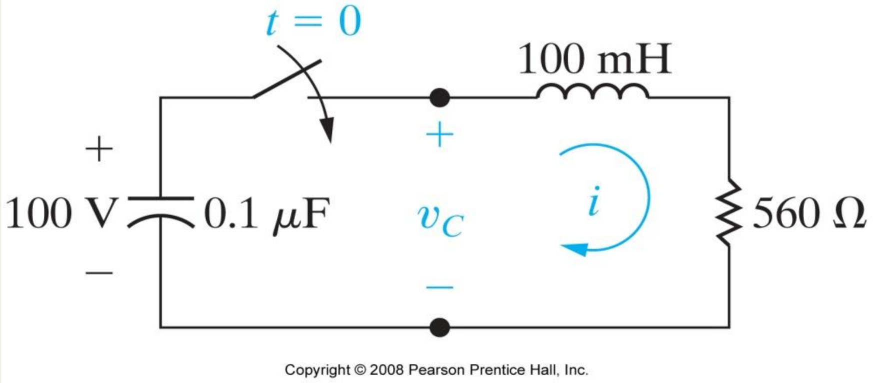
Equation: $i(0) = B_1$ (same as the parallel case!)

Circuit: $i(0) = I_0 = 0$

$$\Rightarrow \quad B_1 = 0$$

Natural Response Series RLC Example

The capacitor is charged to 100 V and at t = 0, the switch closes. Find i(t) for t ≥ 0.



Equation: $$\frac{di(0)}{dt} = -\alpha B_1 + \omega_d B_2 \text{ (same as the parallel case!)}$$

Circuit: $$\frac{di(0)}{dt} = \frac{1}{L} v_L(0) = \frac{1}{L}\left(-v_C(0) - v_R(0)\right) = \boxed{\frac{1}{L}\left(-V_0 - RI_0\right)}$$

$$= \frac{1}{0.1}\left(-(-100) - 560(0)\right) = 1000 \text{ A/s}$$

$$\Rightarrow \quad -2800(0) + 9600B_2 = 1000 \quad \Rightarrow \quad B_2 = 0.104$$

$$\therefore \quad i(t) = 0.104e^{-2800t}\sin 9600t \text{ A}, t \geq 0$$

Natural Response of RLC Circuits – Summary

Use the Second-Order Circuits table:

1. Make sure you are on the Natural Response side.
2. Find the appropriate column for the RLC circuit topology.
3. Make sure the initial conditions are defined exactly as shown in the figure!
4. Use the equations in Row 4 to calculate α and ω_0.
5. Compare the values of α and ω_o to determine the response form (given in one of the last 3 rows).
6. Use the equations to solve for the unknown coefficients.
7. Write the equation for *v(t), t ≥ 0* (parallel) or *i(t), t ≥ 0* (series).
8. Solve for any other quantities requested in the problem.