Condition for maximum power transfer

Suppose we can vary the load resistance. For what value of load resistance will maximum power be absorbed by the load?

$$p_L = i^2 R_L; \qquad i = \frac{V_{Th}}{R_{Th} + R_L}$$

$$\therefore \qquad p_L = \frac{R_L V_{Th}^2}{(R_{Th} + R_L)^2}$$

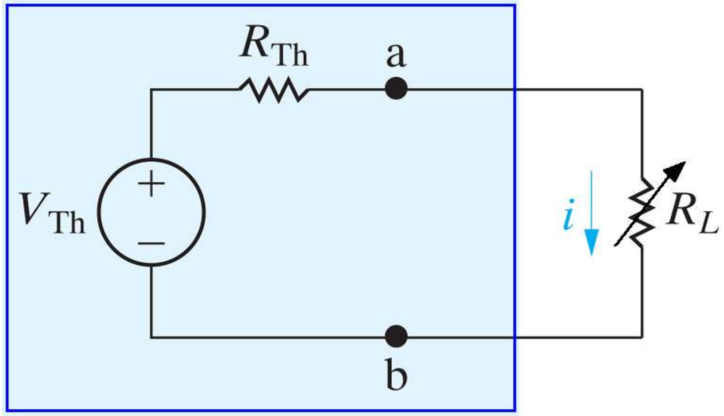
$$\text{For max. power,} \quad \frac{dp_L}{dR_L} = 0$$

$$\frac{dp_L}{dR_L} = \frac{V_{Th}^2}{(R_{Th} + R_L)^2} - \frac{2R_L V_{Th}^2}{(R_{Th} + R_L)^2}$$

$$= \frac{V_{Th}^2 (R_{Th} + R_L) - 2R_L V_{Th}^2}{(R_{Th} + R_L)^3} = 0$$

$$\Rightarrow \qquad V_{Th}^2 (R_{Th} + R_L) - 2R_L V_{Th}^2 = 0$$

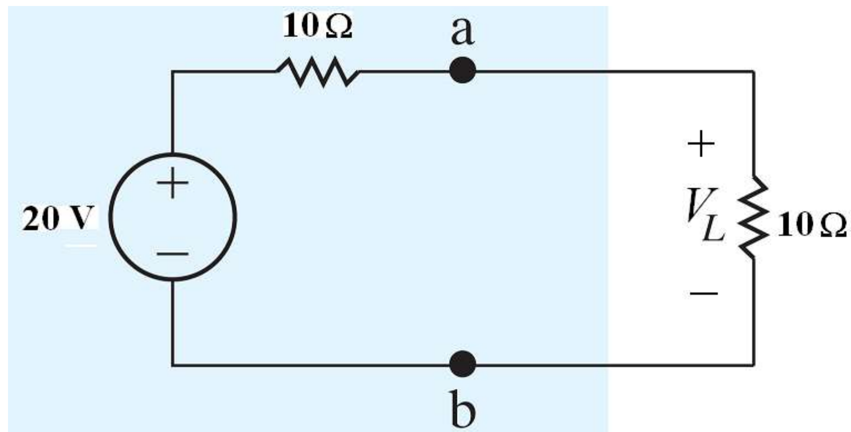
$$\Rightarrow \qquad (R_{Th} + R_L) = 2R_L \qquad \therefore \qquad R_L = R_{Th}$$



Represents the Thevenin equivalent of an arbitrary subcircuit that will not change

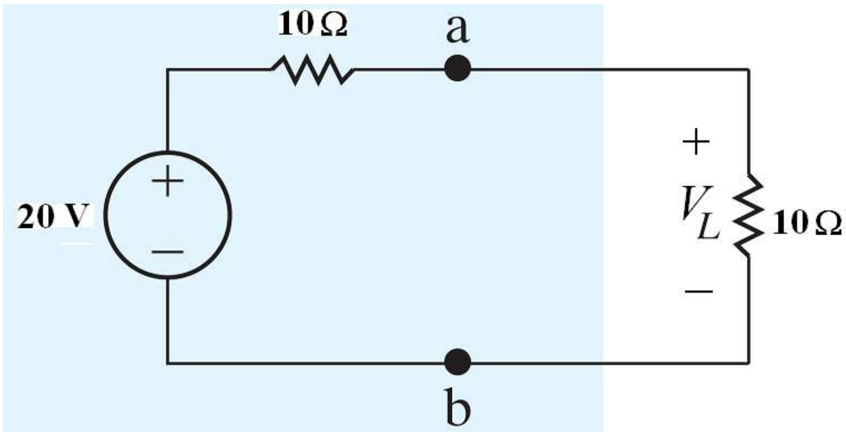
Represents a variable load

In the circuit below, the load resistor is matched to the Thevenin resistance – how much voltage drops across the load resistor?



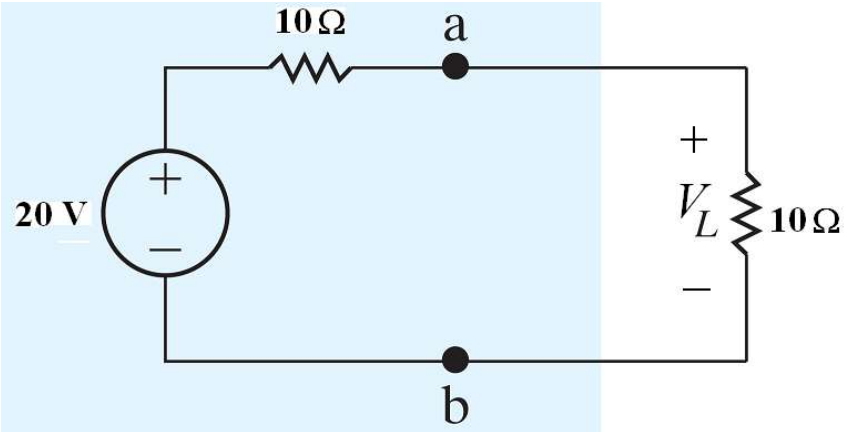
- A. 20 V
- B. 10 V
- C. 5 V
- D. 40 V

In the circuit below, the load resistor is matched to the Thevenin resistance – how much power is absorbed by the load resistor?



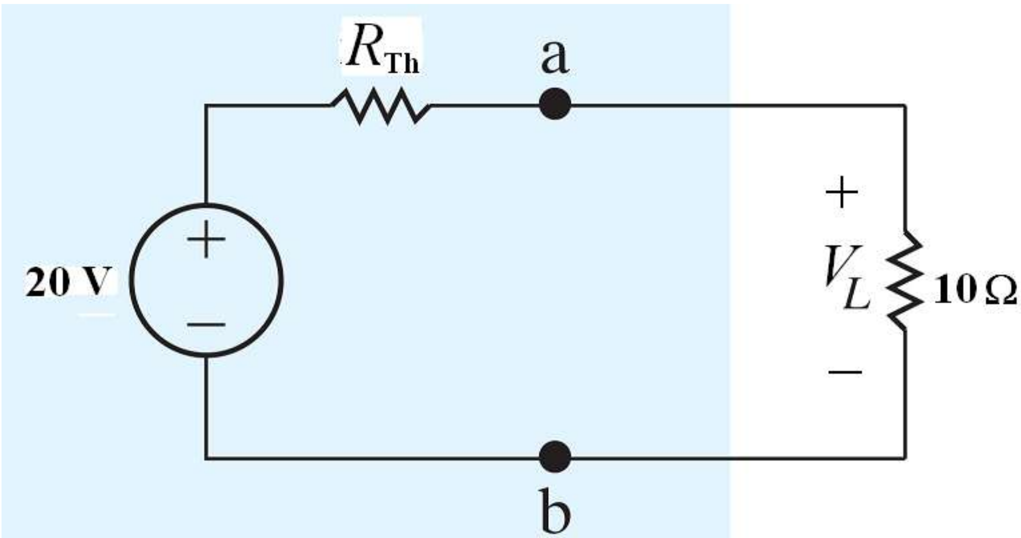
A. 100 W
B. 50 W
C. 20 W
D. 10 W

IN THE CIRCUIT BELOW, THE LOAD RESISTOR IS MATCHED TO THE THEVENIN RESISTANCE – HOW MUCH POWER IS DELIVERED BY THE VOLTAGE SOURCE?



A. 100 W
B. 50 W
C. 20 W ✓
D. 10 W

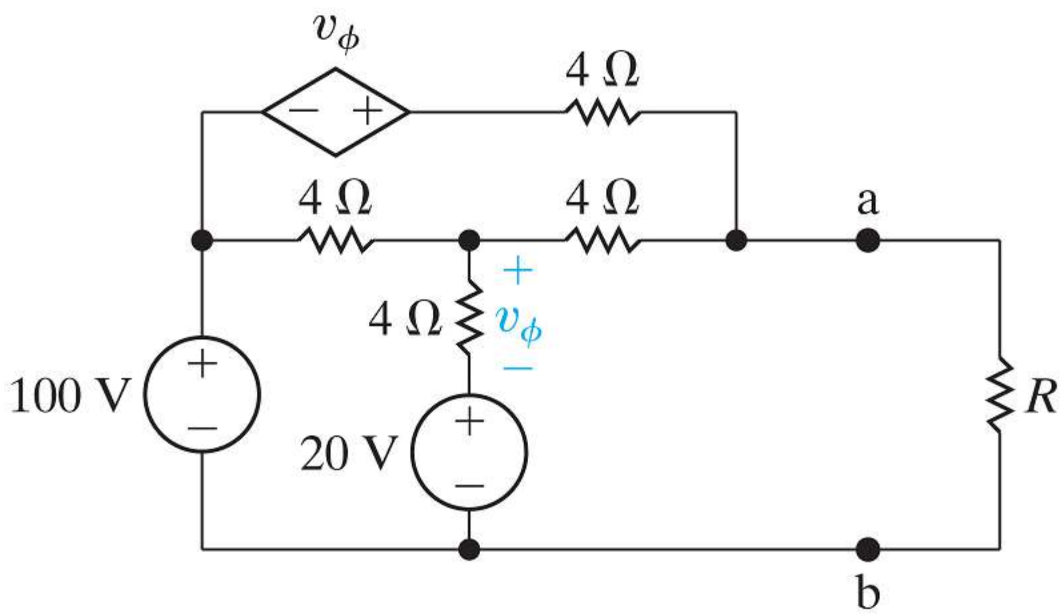
Suppose that we vary the Thevenin resistor instead of the load resistor. For what value of Thevenin resistance is maximum power delivered to the load resistance?



A. 20 Ω
B. 10 Ω
C. 0 Ω
D. None of the above

Example of maximum power transfer

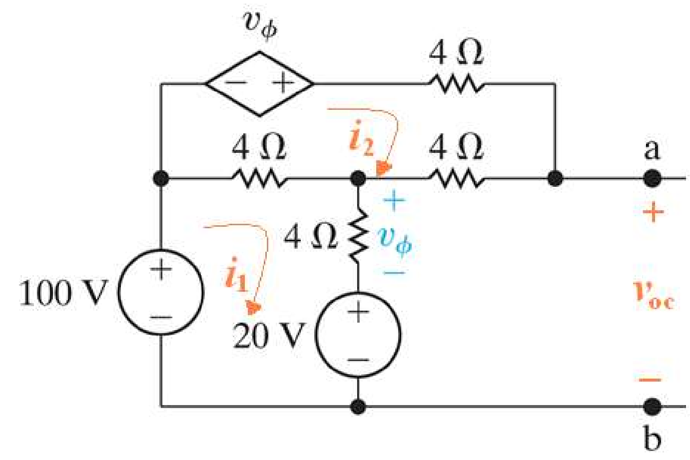
Find the value of R such that maximum power is transferred to R, and find that maximum power.



Approach – find the open circuit and the short circuit current by analyzing two circuits – see the next slides.

Find the value of R such that maximum power is transferred to R, and find that maximum power.

Find the open circuit voltage:



$$i_1 \text{ mesh}: \; -100 + 4(i_1 - i_2) + 4i_1 + 20 = 0$$

$$i_2 \text{ mesh}: \; -v_\phi + 4i_2 + 4(i_2 - i_1) = 0$$

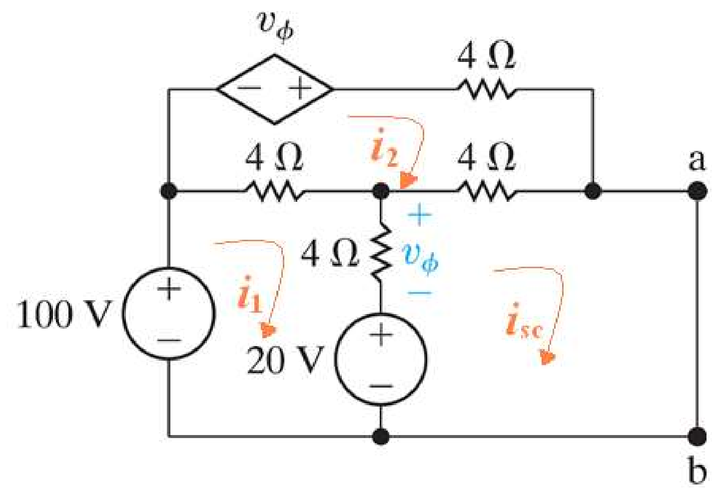
$$\text{constraint}: \; v_\phi = 4i_1$$

$$\text{Solving}: \; i_1 = 15 \text{ A}; \qquad i_2 = 10 \text{ A}; \qquad v_\phi = 60 \text{ V}$$

$$\therefore \qquad v_{oc} = 4i_2 + 4i_1 + 20 = 120 \text{ V}$$

Find the value of R such that maximum power is transferred to R, and find that maximum power.

Find the short circuit current:



$$i_1 \text{ mesh}: -100 + 4(i_1 - i_2) + 4(i_1 - i_{sc}) + 20 = 0$$

$$i_2 \text{ mesh}: -v_\phi + 4i_2 + 4(i_2 - i_{sc}) + 4(i_2 - i_1) = 0$$

$$i_{sc} \text{ mesh}: -20 + 4(i_{sc} + i_1) + 4(i_{sc} - i_2) = 0$$

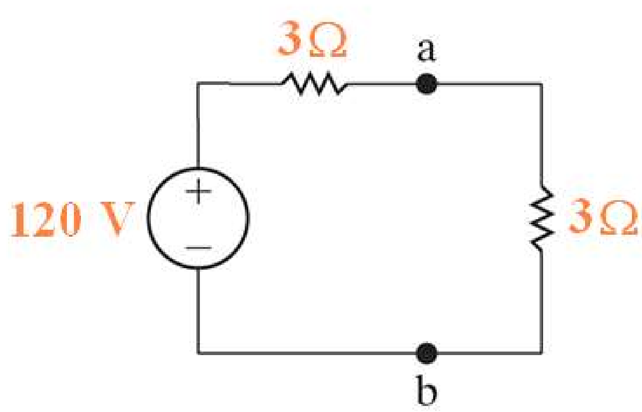
$$\text{constraint}: v_\phi = 4(i_1 - i_{sc})$$

$$\text{Solving}: i_1 = 45\text{ A}; \quad i_2 = 30\text{ A}; \quad i_{sc} = 40\text{ A}; \quad v_\phi = 20\text{ V}$$

$$\therefore \quad R_{Th} = \frac{v_{oc}}{i_{sc}} = \frac{120}{40} = 3\Omega$$

Find the value of R such that maximum power is transferred to R, and find that maximum power.

The Thevenin equivalent:



$$p_L = \frac{60^2}{3} = 1200\text{ W}$$

Chapter 4 – techniques of circuit analysis

Summary of circuit analysis techniques:

- Ohm's Law
- KVL and KCL
- Combining resistors in series and parallel
- Voltage division and current division
- Node voltage method
- Mesh current method
- Source transformation
- Thevenin and Norton equivalents