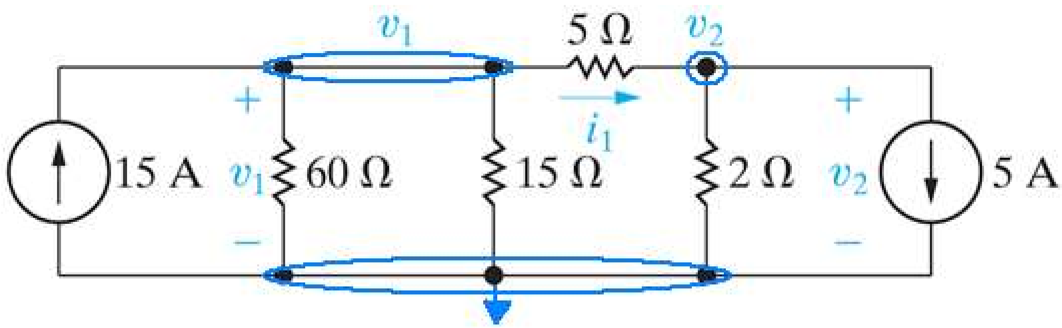
Chapter 4 – techniques of circuit analysis

Node voltage method:

- Based on writing KCL equations at essential nodes
- Solves for node voltages
- The "recipe":
  1. Identify the essential nodes
  2. Pick a reference node
  3. Label remaining essential nodes with voltage values
  4. Write a KCL equation at each non-reference essential node
  5. Put equations in standard form and solve
  6. Check your solutions by balancing power
  7. Calculate quantities of interest

Node voltage method:

Find the two voltages and the current indicated.



$$\text{At } v_1: \; -15 + \frac{v_1}{60} + \frac{v_1}{15} + \frac{v_1 - v_2}{5} = 0$$

$$\text{At } v_2: \; \frac{v_2 - v_1}{5} + \frac{v_2}{2} + 5 = 0$$

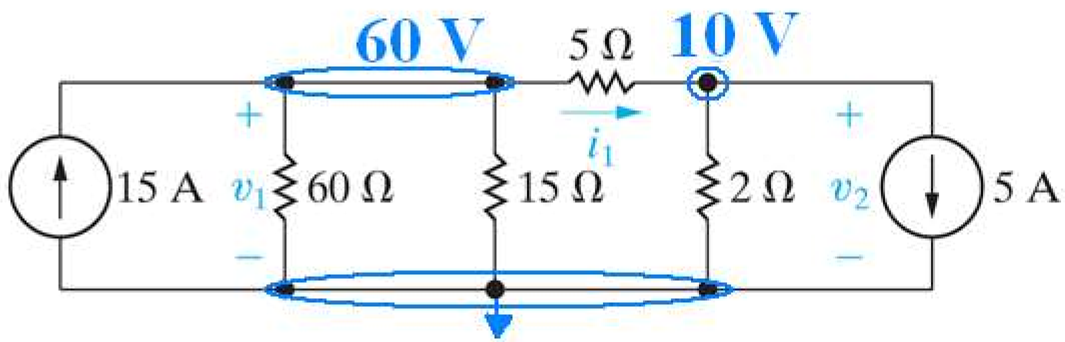
$$\text{Standard form}: v_1\left(\frac{1}{60} + \frac{1}{15} + \frac{1}{5}\right) + v_2\left(-\frac{1}{5}\right) = 15$$

$$v_1\left(-\frac{1}{5}\right) + v_2\left(\frac{1}{5} + \frac{1}{2}\right) = -5$$

$$\text{Calculator solution}: \quad v_1 = 60\text{ V}; \quad v_2 = 10\text{ V}$$

Node voltage method (continued):

Find the two voltages and the current indicated.



Power check :

$p_{15A} = -(15)(60) = -900$ W; $p_{60} = 60^2/60 = 60$ W;

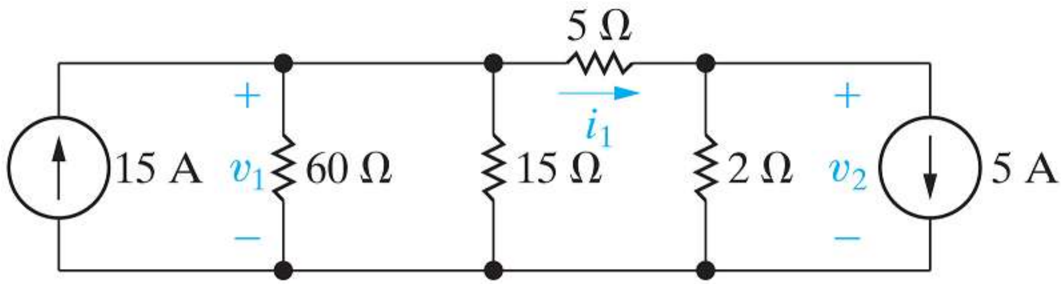
$p_{15\Omega} = 60^2/15 = 240$ W; $p_5 = (60-10)^2/5 = 500$ W;

$p_2 = 10^2/2 = 50$ W; $p_{5A} = (5)(10) = 50$ W

$$\sum p = -900 + 60 + 240 + 500 + 50 + 50 = 0$$

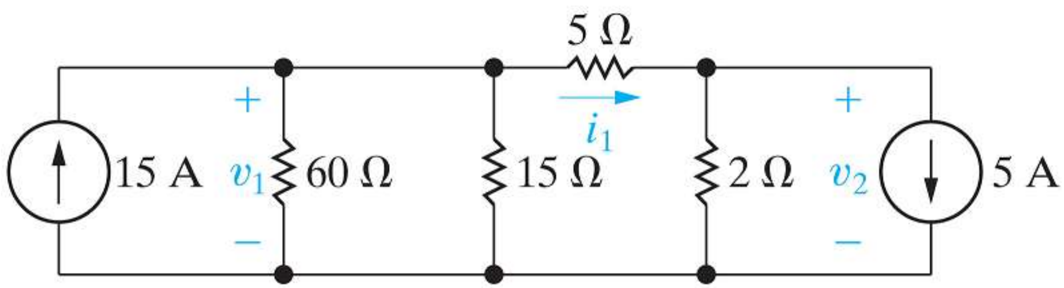
Calculate i_1 : $i_1 = (60-10)/5 = 10$ A

If v_1 is 60 V and v_2 is 10 V, what is the power associated with the 60 Ω resistor?



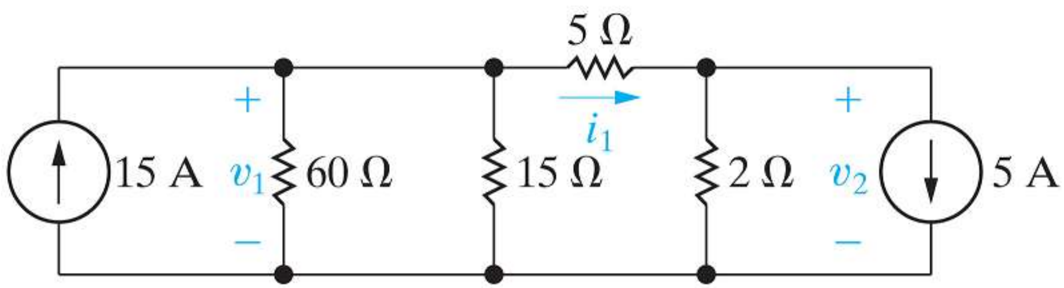
A. 60 W
B. 240 W
C. 3600 W
D. None of the above

If v_1 is 60 V and v_2 is 10 V, what is the power associated with the 5 A source?



A. 900 W
B. –900 W
C. –50 W
D. 50 W

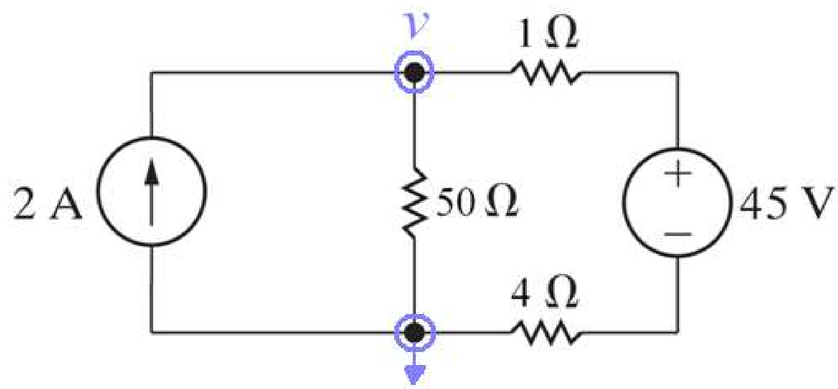
IF v_1 IS 60 V AND v_2 IS 10 V, WHAT IS THE CURRENT i_1?



A. 14 A
B. –10 A
C. 10 A
D. 250 A

Node voltage method:

Find the power associated with the current source.



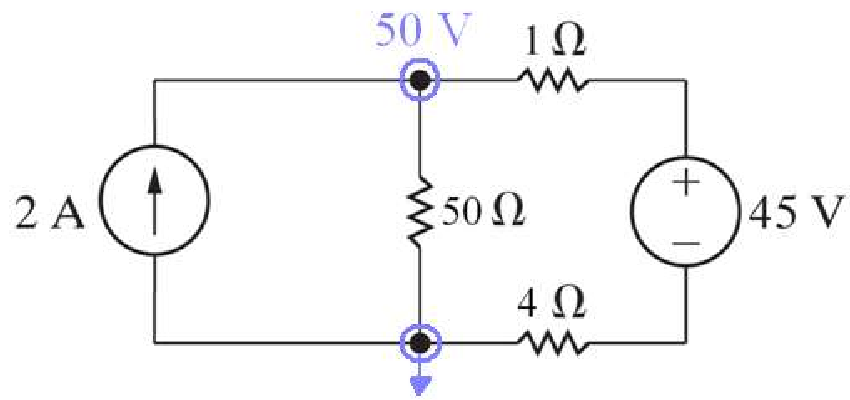
KCL at v: $-2+\frac{v}{50}+\frac{v-45}{1+4}=0$

Standard form: $v\left(\frac{1}{50}+\frac{1}{5}\right)=2+\frac{45}{5}$

Solving: $(50)\left[v\left(\frac{1}{50}+\frac{1}{5}\right)=2+\frac{45}{5}\right]\Rightarrow v(1+10)=100+450$

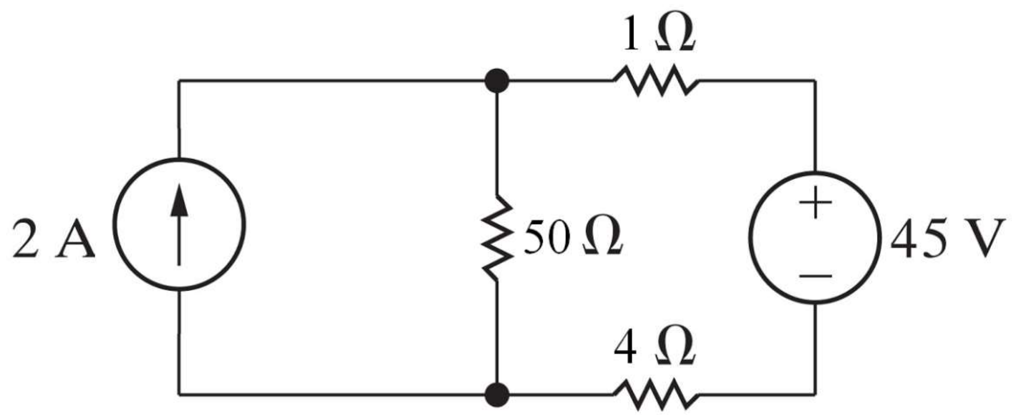
$\therefore \quad v=\frac{550}{11}=50\text{ V}$

Power balance:



Component	Equation	p [W]
2 A	–(2)(50)	–100
45 V	(45)[(50 – 45)/5]	45
1 Ω	$(1)^2(1)$	1
4 Ω	$(1)^2(4)$	4
50 Ω	$(50)^2/50$	50

THE CURRENT SOURCE IS ABSORBING POWER.



A. True

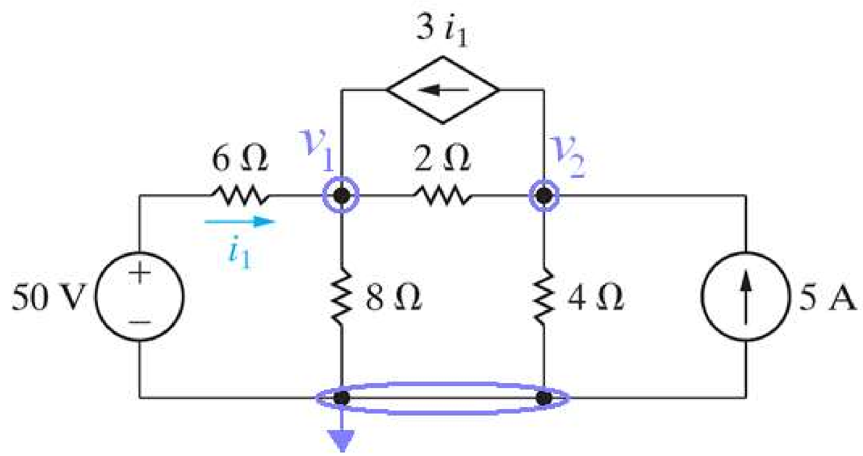
B. False

Node voltage method – special cases:

- The circuit contains one or more dependent sources
- The circuit has a branch that connects a non-reference essential node and the reference node, and this branch contains a voltage source only.
- The circuit has a branch that connects two non-reference essential nodes, and this branch contains a voltage source only.

Node voltage method with dependent sources:

Find the power associated with each source.



$$\text{KCL at } v_1: \frac{v_1 - 50}{6} + \frac{v_1}{8} + \frac{v_1 - v_2}{2} - 3i_1 = 0$$

$$\text{KCL at } v_2: \frac{v_2 - v_1}{2} + \frac{v_2}{4} - 5 + 3i_1 = 0$$

THE TWO NODE VOLTAGE EQUATIONS WE JUST CONSTRUCTED ARE GIVEN BELOW. HOW MANY UNKNOWNS DO THESE EQUATIONS CONTAIN?

$$\frac{v_1 - 50}{6} + \frac{v_1}{8} + \frac{v_1 - v_2}{2} - 3i_1 = 0$$

$$\frac{v_2 - v_1}{2} + \frac{v_2}{4} - 5 + 3i_1 = 0$$

- ✗ A. 2
- ✓ B. 3
- ✗ C. 4
- ✗ D. None of the above

Node voltage method with dependent sources:

- The “modified recipe”:
  1. Identify the essential nodes
  2. Pick a reference node
  3. Label remaining essential nodes with voltage values
  4. Write a KCL equation at each non-reference essential node
     a) Any dependent sources? If so, write a constraint equation for each one that defines the variable the source depends upon
  5. Put equations in standard form and solve
  6. Check your solutions by balancing power
  7. Calculate quantities of interest

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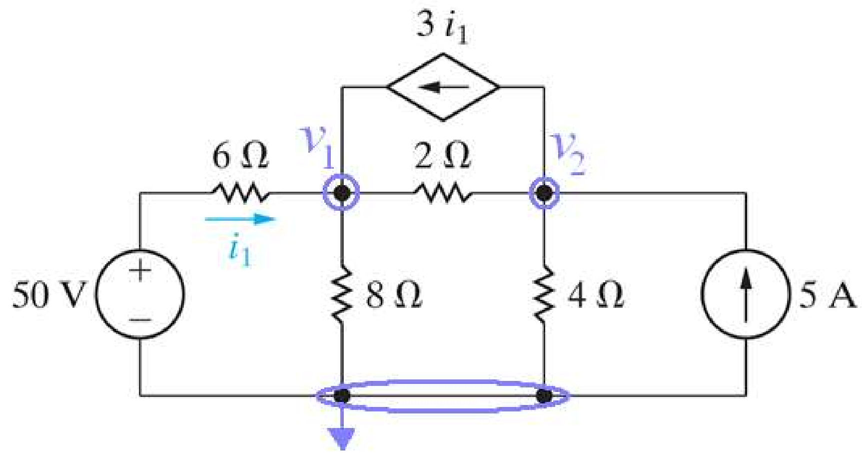
Node voltage method with dependent sources:

Continue with the dependent source constraint equation.



$$\text{KCL at } v_1: \frac{v_1-50}{6}+\frac{v_1}{8}+\frac{v_1-v_2}{2}-3i_1=0 \qquad \text{KCL at } v_2: \frac{v_2-v_1}{2}+\frac{v_2}{4}-5+3i_1=0$$

$$\text{dep. source constraint}: i_1=\frac{50-v_1}{6}$$

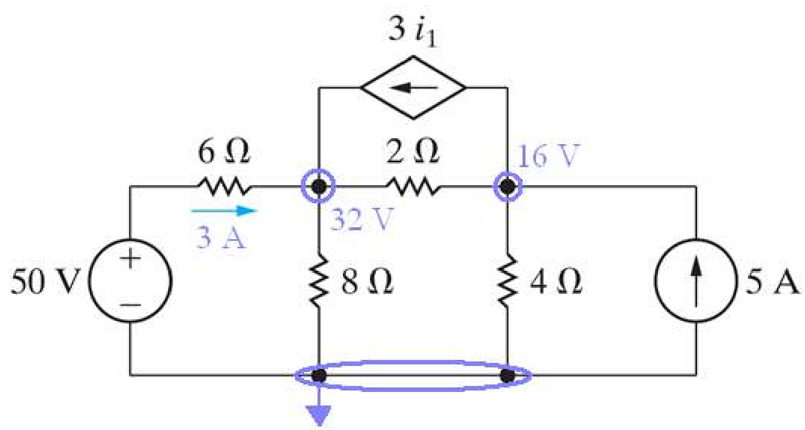
$$v_1(1/6+1/8+1/2) \quad +v_2(-1/2) \quad +i_1(-3)=50/6$$

$$\text{Std form}: \quad v_1(-1/2) \quad +v_2(1/2+1/4) \quad +i_1(3)=5$$

$$v_1(1/6) \quad +v_2(0) \quad +i_1(1)=50/6$$

$$\text{Solving}: v_1=32\text{ V}; \quad v_1=16\text{ V}; \quad i_1=3\text{ A}$$

Power balance:



Component	Equation	p [W]
50 V	–(3)(50)	–150
5 A	–(5)(16)	–80
Dep. Source	–[3(3)](32 – 16)	–144
6 Ω	$(3)^2(6)$	54
2 Ω	$(32 – 16)^2/2$	128
8 Ω	$(32)^2/8$	128
4 Ω	$(16)^2/4$	64